Abstract. The effects of solid-solid phase changes on subsolidus convection in the large icy moons of the outer solar system are considered. Phase transitions affect convection via processes that distort the phase change boundary and/or influence buoyancy through thermal expansion. Linear stability analyses are carried out for ice layers with a phase change at the midplane. Two exothermic phase transitions (ice I - ice II, ice VI - ice VIII) and two endothermic transitions (ice I - ice III, ice II - ice V) are considered. For the exothermic cases, the phase change can either impede or enhance whole-layer convection. For the endothermic cases, the phase change always inhibits whole-layer convective overturn and trends to enforce two-layer convection. These results place some constraints on possible models of icy satellite evolution and structure.

Introduction

The role of subsolidus convection in the icy Galilean and Saturnian satellites has inspired a good deal of discussion, both before [Reynolds and Cassen, 1979], and after [Parmentier and Head, 1979; Thurber et al., 1980; Schubert et al., 1981, 1986; Cassen et al., 1981; Ellsworth and Schubert, 1983], the imaging experiments of Voyagers 1 and 2 [Smith et al., 1979a, b, 1981, 1982] revealed resurfacing and tectonic activity on several of these moons. However, the influence of ice phase changes on convection has received relatively little attention. Thurber et al. [1980] considered the ice II - ice V phase change and concluded that it impeded convection. Reynolds et al. [1981] investigated the effects on convection of several ice phase changes, both exothermic and endothermic. They concluded that exothermic transitions are unstable to convection for a wide range of temperature gradients and that endothermic transitions are also unstable to convection under conditions of high heat flow; the endothermic transitions are stable against convective motion for lower heat flows that correspond to later times in an icy moon's evolution. In their thermal model of Ganymede, Kirk and Stevenson [1983] proposed that the ice I - ice III phase change is unstable to convection and would thereby initiate whole-layer overturn within the outer strata of Ganymede early in its evolution.

In this paper, we examine the interaction of subsolidus convection in ice with two exothermic phase transitions (ice I - ice II, and ice VI - ice VIII) and with two endothermic ones (ice I - ice III, and ice II - ice V). We carry out linear stability analyses following the approach of Schubert et al. [1975] for the olivine-spinel (exothermic) and the spinel-postspinel (endothermic) transitions in the earth's mantle. By determining whether the specified ice phase changes either enhance or impede convection we can constrain evolutionary and structural models of the icy Jovian and Saturnian moons. In particular, we find that, as an icy moon cools, the exothermic phase transitions change from being stabilizing to destabilizing, whereas the endothermic phase changes are always stabilizing against convection.

Phase Change Effects

Schubert et al. [1975] have discussed the physics of the interaction of solid-solid phase changes with convection. Two effects, advection of the ambient temperature and release or absorption of latent heat, distort the phase boundary. The distortion causes a hydrostatic pressure head which enhances or impedes convection across the boundary depending on the sign of the pressure head. The third effect is also due to latent heat which influences the material's buoyancy through ordinary thermal expansion.

The advection of the ambient temperature distorts the phase boundary via temperature perturbations induced by upwelling and downwelling fluid. When hot upwelling material rises through an exothermic (or endothermic) phase change the boundary is distorted vertically since it must always lie along the Clapeyron curve. The increase in temperature causes the boundary to distort downwards to higher pressures if it is exothermic (with a Clapeyron curve whose slope is positive) and upwards if endothermic (negative slope). Therefore, a hydrostatic head is developed which either enhances (for exothermic) or stabilizes against (for endothermic) upward flow. Similarly, downwelling, cold material causes the boundary to distort upwards (exothermic) or downwards (endothermic) which affects the stability of the flow similarly to the upwelling case.

The latent heat also distorts the boundary by increasing or decreasing the local temperature. Upwelling material will, upon passing through the equilibrium boundary, either lose heat (exothermic) or gain heat (endothermic). By the nature of the Clapeyron curve, both cases will distort the boundary upwards, creating a hydrostatic head which
TABLE 1. Properties of Ice at the Temperatures and Pressures of the Listed Phase Changes

<table>
<thead>
<tr>
<th></th>
<th>I - II</th>
<th>I - III</th>
<th>II - V</th>
<th>VI - VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (*K$^{-1}$)</td>
<td>1.25x10$^{-4}$</td>
<td>1.4x10$^{-4}$</td>
<td>1.4x10$^{-4}$</td>
<td>1.6x10$^{-4}$</td>
</tr>
<tr>
<td>$g$ (m s$^{-1}$)</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>$D$ (km)</td>
<td>150</td>
<td>80</td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>$\kappa$ (m$^2$ s$^{-1}$)</td>
<td>1.45x10$^{-6}$</td>
<td>9.2x10$^{-7}$</td>
<td>6.24x10$^{-7}$</td>
<td>4.2x10$^{-7}$</td>
</tr>
<tr>
<td>$\gamma$ (Pa K$^{-1}$)</td>
<td>8.1x10$^{5}$</td>
<td>-2x10$^{6}$ → -5x10$^{5}$</td>
<td>-6.73x10$^{5}$</td>
<td>3.5x10$^{6}$</td>
</tr>
<tr>
<td>$\Delta\rho$ (kg m$^{-3}$)</td>
<td>240</td>
<td>204</td>
<td>61</td>
<td>~ 200</td>
</tr>
<tr>
<td>$\rho$ (*kg m$^{-3}$)</td>
<td>1063</td>
<td>1042</td>
<td>1236</td>
<td>~ 1700</td>
</tr>
<tr>
<td>$Q/10^2$ (J kg$^{-1}$)</td>
<td>41.8 → 34.8</td>
<td>-0.987xT + 225.9</td>
<td>-66.9 → -64.4</td>
<td>65.5</td>
</tr>
<tr>
<td>$c_p/10^2$ (J kg$^{-1}$K$^{-1}$)</td>
<td>1.84 → 1.51</td>
<td>0.0081xT - 0.094</td>
<td>1.92 → 1.84</td>
<td>2.16</td>
</tr>
<tr>
<td>$Q/c_p$ (K)</td>
<td>23</td>
<td>---</td>
<td>-34.9</td>
<td>30.3</td>
</tr>
</tbody>
</table>

Temperatures and pressures lie along the Clapeyron curves of the respective phase changes. A range or function is indicated when a variation occurs along the Clapeyron curve.


inhibits upward flow. The distortion of the boundary for downwelling material is in the opposite sense and also impedes flow across the phase boundary. Therefore, the distortion of the equilibrium boundary from latent heat is wholly stabilizing against convection.

Finally, latent heat will contribute a stabilizing (exothermic) or destabilizing (endothermic) influence on convection through thermal expansion. Regardless of phase boundary distortions, upwelling material will heat up with an endothermic boundary, becoming less stable, or cool off for an exothermic boundary, becoming more stable. Again, downwelling material will be influenced identically to upwelling material with respect to stability.

To summarize the effects of phase changes, one can see that for exothermic boundaries the latent heat is stabilizing against convection in both manners of influence (boundary distortion and thermal expansion) while the advection of the ambient temperature is destabilizing. For an endothermic transition, the distortion of the boundary (for latent heat or temperature advection) is entirely stabilizing while the influence on thermal expansion by latent heat is destabilizing. It is apparent that the effect of phase changes on convective instability depends on several competing processes.

Linear Stability Analysis

The development of a linearized theory of stability for a fluid with a univariant phase change and shear stress-free boundaries was done by Schubert and Turcotte [1971]. The results of the stability analysis can be presented as a plot of the minimum super-adiabatic temperature gradient $(\beta - \beta_a)_{crit}$ at the onset of convection versus kinematic viscosity $\nu$. $(\beta$ is the ambient temperature gradient and $\beta_a$ is the adiabatic temperature gradient). This may be compared to similar plots for Rayleigh-Bénard (R-B) convection with no phase change for either single

![Fig. 1. Viscosity $\eta$ vs. strain rate $\dot{e}$ for ice I (dashed) and ice II (solid) at 200 K and ice I (dashed) and ice III (solid) at 250 K. Rheological laws from Durham et al. [1985] and Kirby et al. [1985].](image-url)
entering into the stability criterion to the fourth power. In this model, D of each layer is characteristic of a differentiated, static, conducting moon the size of Ganymede with temperatures corresponding to conditions after a liquid water mantle has frozen out. Although the stability analysis is carried out only for temperatures at which the considered phases exist, one must bear in mind that the thicknesses of certain ice phases, namely ice III and ice V, will change markedly as the moon cools.

Kinematic viscosity is calculated using the Arrhenius law \[\nu = 0.139\exp(8750/T)\]

where T is in Kelvins. Recent investigations [Kirby et al., 1984; Durham et al., 1985] have developed rheological laws for ices I, II, and III. These studies demonstrated that for laboratory strain rates greater than about \(10^{-4}\) s\(^{-1}\), ice III is weaker than ice I, which in turn is weaker than ice II. However, when these laws are used to calculate effective viscosities at geological strain rates, one finds that the viscosities for any two of these ice phases which co-exist at the same temperature are similar (Figure 1). Thus, although there are presently only limited laboratory data on the rheological behavior of ices V, VI and VIII, we use a single viscosity law for the different phases.

The viscosities along each of the curves in Figures 2 and 3 are calculated using (1) with temperatures that lie along the Clapeyron curve of each phase change. This assumes the occurrence of phase transitions at equilibrium temperatures. However, below certain temperatures, the reaction rates of most phase changes are exceedingly slow [Bridgeman, 1912], and a phase may exist metastably in the stability field of an adjacent phase.

The \((\beta - \beta_a)_{\text{crit}}\) versus \(\nu\) curves for the

![Fig. 2. Minimum superadiabatic temperature gradient \((\beta - \beta_a)_{\text{crit}}\) for the onset of convection versus kinematic viscosity \(\nu\) for exothermic phase transitions ice I - ice II and ice VI - ice VIII. Viscosity and other parameters are calculated for the ices at temperatures and pressures on the respective Clapeyron curves (inset phase diagram). The bold lines represent stability curves for single-cell (solid) and double-cell (dashed) Rayleigh-Bénard convection in ice without a phase change. The thin solid curves represent the stability curves for ice with a phase change. For a given viscosity, the curve with the smallest value of \((\beta - \beta_a)_{\text{crit}}\) represents the most destabilizing mechanism. A-B and C-D on the phase diagram delineate portions of the equilibrium phase boundaries along which the stability curves are computed.

or double layer overturn (e.g., Figures 2 and 3). For a given viscosity, the case with the smallest value of \((\beta - \beta_a)_{\text{crit}}\) is the least stable. Thus, if the curve of the fluid with a phase change lies above the double-cell R-B curve, it is more stable than two-layer convection and thereby inhibits whole-layer overturn. If the curve lies between the two R-B lines, the phase change allows single-cell convection but the fluid would convect more readily if there were no phase change. When the curve is below the single-cell R-B line, the phase transition enhances whole-layer overturn. Thus, from this comparison, one may perceive the stability of a fluid with a phase change relative to the same fluid without a phase change.

**Results**

Table 1 gives the values of mechanical and thermal parameters used in the stability analyses. Most of the numerical constants do not vary greatly with temperature. However, the relevant layer thickness D is strongly dependent on the thermal evolution of a satellite (e.g., the ice III and ice V layers disappear completely at low temperatures). Therefore, D is the most uncertain of the parameters listed in Table 1, but, it is also one of the most influential,
The phase changes in ice are complex and depend on temperature and pressure. For viscosities greater than about $3 \times 10^{-17}$ m$^2$ s$^{-1}$ (ice I - ice II) or $2 \times 10^{-17}$ m$^2$ s$^{-1}$ (ice VI - ice VIII), the phase changes in fact enhance single layer convection. For the ice II - ice V transition, the curve approaches single layer convective instability for viscosities around $10^{-16}$ m$^2$ s$^{-1}$.

Discussion

The effects of phase transitions on convective instability may have a profound influence on the thermal histories of the larger icy moons. Early in the evolution of a differentiated ice moon, after a water mantle between the ice I and ice III layers has been frozen out, 2-layer convection will likely be preferred with the ice I - ice III boundary acting as a barrier to whole layer overturn. When the temperature of the moon decreases to the point where the ice I - ice II transition exists, 2-layer convection will probably still occur. However, as the temperature continues to decrease, the phase change will allow single layer convection and as the temperature nears approximately 200K, the phase change will enhance whole layer overturn. Although these low temperatures are in the region of very slow reaction rates, the rate of overturn is so small that the phase boundary will likely be maintained at its equilibrium position.

For the larger moons that may contain ice polymorphs of density higher than that of ice II, the ice V - ice VI phase change always inhibits single layer convection. At depths near possible silicate cores in these moons, the ice VI - ice VIII phase transition will impede single layer overturn while the moon is still relatively warm. As the moon cools, however, single layer convection will be favored and eventually, at even colder temperatures, the phase change will enhance whole layer convection.

References


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