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On the Purpose of Toroidal Motion in a Convecting Mantle

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Abstract. The purpose of toroidal flow, i.e., strike-slip motion and plate spin, in the plate-tectonic style of mantle convection is enigmatic. It is a purely horizontal, dissipative flow field that makes no apparent contribution to the release of heat. However, when plate-like toroidal motion is allowed to arise as a phenomenon of non-Newtonian mantle dynamics, it in fact acts to reduce the net amount of viscous dissipation. We show that for a non-Newtonian flow driven by an existing poloidal field (a source-sink field), the generation of toroidal motion interacts with the nonlinear rheology to cause less viscous dissipation than if there were no toroidal motion. With power-law rheologies, the generation of toroidal motion causes up to a 25% reduction in viscous dissipation. With a self-lubricating rheology, which has been shown to induce the most plate-like behavior, toroidal motion causes as much as an 80% reduction in viscous dissipation. Thus, basic non-Newtonian fluid dynamical theory of the formation of plate tectonics shows that toroidal motion is far from superfluous but in fact facilitates the efficiency of convective flow at the surface.

Introduction

One of the most fundamental goals of geodynamics is to understand how plate tectonics arises as a convective self-organizing structure from the lithosphere-mantle system. A defining feature of the plate-tectonic style of mantle convection is the existence of toroidal flow, i.e., motion along transform faults, oblique subduction and in the bulk spin of plates. Such motion at the Earth's surface has almost as much kinetic energy as the poloidal part which is manifest at the surface in convergent and divergent motion [Hager and O'Connell, 1978, 1979, 1981; O'Connell et al., 1991; Cadek and Ricard, 1992; Lithgow-Bertelloni et al., 1993]. Poloidal motion throughout the mantle is associated with upwellings and downwellings; it is linked to heat transport and directly driven by buoyancy forces. Toroidal motion, on the other hand, is apparently superfluous. Since it derives and dissipates energy from the poloidal flow [see also Forte and Peltier, 1987; Gable et al., 1991; Ribe, 1992], and only involves horizontal motion, it would not seem to facilitate efficient release of gravitational potential energy or heat. Thus, the purpose of such motion in a convecting medium such as the plate-mantle system is enigmatic.

Non-Newtonian fluid dynamical theory of plate generation [e.g., Ribe, 1992; Weinstein and Olson, 1992; Bercovici 1993, 1995], however, can be used to show that toroidal motion in fact serves a very important purpose. In this paper we demonstrate that the generation of the toroidal flow field acts to reduce the net viscous dissipation of the convecting system, in the top thermal boundary layer or lithosphere, in particular. If toroidal motion arises from the interaction of a non-Newtonian or nonlinear rheology with the convective (i.e., poloidal) flow, then the localization of strike-slip shear into narrow zones with low viscosity can greatly reduce the dissipation of work done to move material from a source (a spreading center) to a sink (a convergent or subduction zone). Thus, although the toroidal field is added onto a pre-existing poloidal field, it acts in concert with the nonlinear plate-mantle rheology to lubricate the mass transfer of material from divergent to convergent zones.

The non-Newtonian source-sink model

One of the simplest theories describing how toroidal motion arises as a fluid dynamical phenomenon from the interaction of nonlinear rheology and convective flow was proposed by Bercovici [1993, 1995]. In this model, shallow-layer lithospheric motion is driven by sources and sinks which are a proxy for convective motion in that they represent spreading centers and subduction zones, respectively.

The toroidal flow field is then found through the solution of the equations for non-Newtonian viscous flow forced by this source-sink field. This model was used to infer the non-Newtonian rheology which yielded the most plate-like toroidal flow field and viscosity distribution. A suite of rheologies were tested for flows driven by source-sink fields derived from both idealized plate motion and the Earth's present day plate motions. The rheology which yields the most plate-like behavior is a self-lubricating rheology which has also been used as a continuum model of stick-slip behavior [Whitehead and Gans, 1974]. Mantle-type power-law rheologies were found inadequate to generate plate-like behavior, even up to extremely high power-law indices [see also Christensen and Harder, 1991; Cadek et al., 1993]. Extensive discussion of the source-sink model and the survey of rheologies can be found in Bercovici [1993, 1995]. In this paper, we will use this model to demonstrate that the excitation of toroidal motion in non-Newtonian lithospheric flow occurs to reduce viscous dissipation.

Viscous dissipation and toroidal flow

In the non-Newtonian source-sink model, lithospheric motion is treated as shallow-layer flow with a horizontal velocity that is separable into poloidal and toroidal parts:

$$\underline{v}_h = \nabla_h \Phi + \nabla \times (\Psi \hat{n}) \tag{1}$$

where Φ is a poloidal scalar potential, $\Psi\hat{n}$ is the toroidal vector potential, ∇_h is the horizontal or lateral gradient, i.e., perpendicular to the vertical unit normal $\hat{n}$ which is $\hat{z}$ in Cartesian geometry and $\hat{r}$ in spherical geometry. The poloidal potential Φ is prescribed by the source-sink field or horizontal divergence D wherein

$$D = \nabla_h \cdot \underline{v}_h = \nabla_h^2 \Phi. \tag{2}$$

The toroidal potential Ψ is determined by solution of the Stokes flow equation with non-Newtonian viscosity [Bercovici, 1993, 1995; see also England and McKenzie, 1982].

We define the normalized net viscous dissipation in the fluid layer as

$$Q_v = \frac{\int_S 2\eta(\dot{e}^2)\dot{e}^2 dS}{\int_S 2\eta(\dot{e}_p^2)\dot{e}_p^2 dS} \tag{3}$$

where S is the horizontal surface area of the layer, $\dot{e}^2$ is the second strain-rate invariant [see Bercovici 1993, 1995 for the exact forms], and $\dot{e}_p^2$ is the poloidal second strain-rate invariant (i.e., $\dot{e}^2$ with $\Psi = 0$). The non-Newtonian viscosity is described by a modified Carreau equation [see Bird et al., 1987]

$$\eta(\dot{e}^2) = A\left(\gamma + \dot{e}^2\right)^{\frac{1-n}{2n}} \tag{4}$$

where A and γ are constants and n is the power-law index. For $n > 0$ the basic power law rheology is obtained. For $n < 0$ a a self-lubricating rheology occurs; in particular, the continuum stick-slip rheology of Whitehead and Gans [1974] is obtained for $n = -1$.

The value of Q_v measures relative viscous dissipation as toroidal motion arises from the interaction of poloidal (convective) flow and non-Newtonian rheology. (Of course, in highly viscous flows the toroidal field arises very rapidly toward its final steady state, which is the solution to the Stokes equation.) At first, before toroidal motion occurs, $Q_v = 1$; the final state, with an established steady toroidal field, will give a different Q_v. Here, we show only the final value of Q_v. An advantage of employing a normalized dissipation is that, as defined, Q_v is independent of the viscosity constant A, the layer thickness and other extensive properties; it also facilitates comparison between cases with different n which have very different viscosity contrasts. Finally, another interpretation of Q_v is that it contains information about the correlation between η and $\dot{e}^2$ (i.e., whether and how viscosity anomalies are correlated with regions of high strain-rate).

In Figure 1 we show Q_v versus n for three different source-sink fields. The first source-sink field (Figure 1a) is derived from the divergence D of the motion of a simple square plate in a Cartesian geometry (the source is the divergence rate at the trailing edge, the sink is the convergence rate at the leading edge). The second source-sink field (Figure 1b) is derived from the surface divergence of a continuous model of the Earth's present day plate motions [Bercovici and Wessel, 1994] as used by Bercovici [1995]. The sources and sinks of these first two cases are intrinsically plate-like (i.e., they are long and narrow and have abruptly truncated ends) since they are derived from plate velocity fields. These source-sink fields are therefore liable to facilitate plate-like toroidal fields for flows with $n \neq 1$. Thus, for comparison, we also show Q_v for a flow driven by a distinctly unplate-like source-sink field (Figure 1c). In this case, the Cartesian theory of Bercovici [1993] is used, but the souce-sink field is essentially dipolar; i.e., D is composed of an axisymmetric, Gaussian-shaped source and an identically shaped sink. The source and sink each have amplitude of 1 (though are obviously opposite in sign), half-widths of 0.25, and are separated by a distance of $\sqrt{2}$ (in the dimensionless units defined in Bercovici [1993]).

Although the toroidal fields generated in the non-Newtonian flows are superimposed on the poloidal fields, they cause a reduction in viscous dissipation, i.e., $Q_v < 1$, for all source-sink fields. For power-law rheologies, toroidal flow reduces viscous heating as much as 25% (depending on n and the source-sink field). For the self-lubricating rheology ($n = -1$), viscous dissipation is reduced between ap-

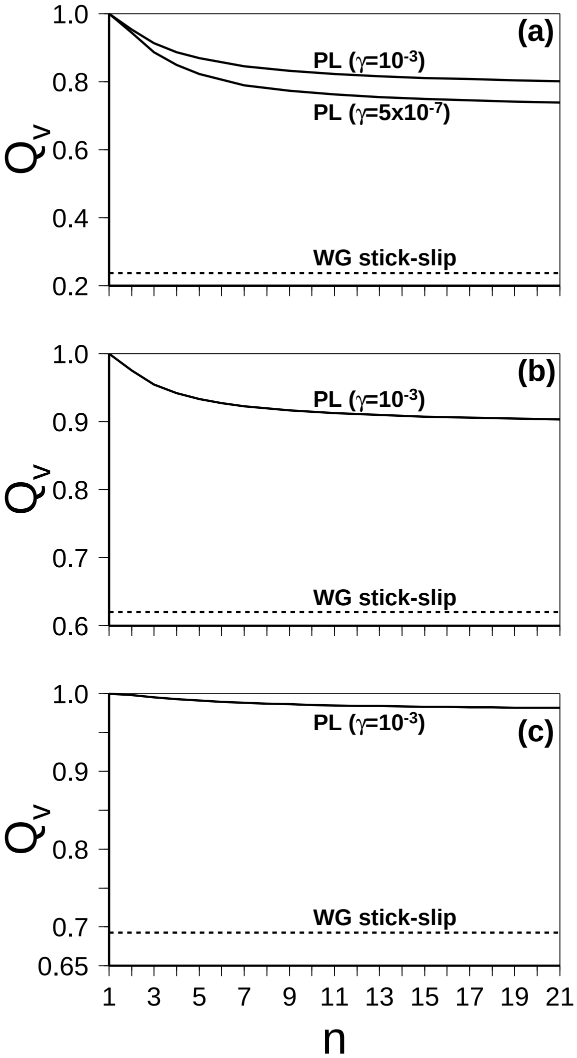

Figure 1. Normalized viscous dissipation Q_v versus power-law index n for non-Newtonian flows driven by (a) the source-sink field derived from the idealized square-plate motion in Cartesian geometry of Bercovici [1993]; (b) the source-sink field derived from the continuous model of Earth's present-day plate-motions [Bercovici and Wessel, 1994] as used by Bercovici [1995]; and (c) an unplate-like, dipolar source-sink field (see text for details). The dashed line indicates the value of Q_v for $n = -1$, i.e., the self-lubricating stick-slip (WG) rheology of Whitehead and Gans [1974]; for this case, $\gamma = 10^{-3}$. Values of γ are shown for the power-law cases (solid lines). Two power-law curves are shown for the Cartesian square-plate case (a) for different values of γ; the smaller value of γ is the same as for the WG stick-slip case, the larger value yields the same maximum viscosity contrast as the WG stick-slip case.

proximatey 30 and 80% (again depending on the source-sink field). Reduction in Q_v is less for the present-day plate tectonic motions than for the square plate. This occurs because for perfect plate motion, the square plate's kinetic energy is 50% toroidal [Olson and Bercovici, 1991; Bercovici, 1993]. However, for the plate-tectonic model, net rotation is removed [see Bercovici, 1995] causing a predominance of poloidal motion [Lithgow-Bertelloni et al., 1993], and hence toroidal motion has less impact on Q_v than for the square-plate model. For the dipolar source-sink field, Q_v is reduced less than for the two plate-like fields, but reduced nonetheless. This occurs because the dipolar source-sink field simply generates less toroidal motion than a plate-like field. However, this demonstrates that a plate-like source-sink field facilitates reduction in viscous dissipation, suggesting that it is not only thermodynamically advantageous for toroidal motion to be generated, but for the poloidal field to be plate-like, as well.

The variational principle for non-Newtonian viscous flows

For non-Newtonian creeping flows, viscous dissipation is not an action variable (unlike the Newtonian case); i.e., it does not obey a principle of least action, or a variational principle wherein it is minimized by the velocity and pressure fields [Bird and Yuen, 1979; Bird et al., 1987]. For incompressible, non-Newtonian Stokes flows in a periodic or enclosed domain V, the action variable minimized by the flow solution is in fact

$$E = 2 \int_V \left(\int_0^{\dot{e}^2} \eta(\dot{\epsilon}^2) d\dot{\epsilon}^2 \right) dV \tag{5}$$

[see Bird et al., 1987 and references therein] which has the same dimensions as viscous dissipation. For equation (4), this quantity is

$$E = \int_V \left\{ \begin{array}{ll} \frac{4An\gamma^{\frac{1+n}{2n}}}{1+n} \left[(1+\dot{e}^2/\gamma)^{\frac{1+n}{2n}} - 1 \right] & n \neq -1 \\ 2A \log(1+\dot{e}^2/\gamma) & n = -1 \end{array} \right\} dV \tag{6}$$

which is indeed the net viscous dissipation for $n = 1$. Although we could use this quantity instead of viscous dissipation, the basic results remain unchanged: toroidal motion reduces E as well as viscous dissipation. We opt to discuss viscous dissipation for the sake of appealing to physical and thermodynamic intuition. However, we can use this least-action principle to shed some light on why toroidal flow reduces viscous dissipation in our non-Newtonian fluid dynamical model of plate tectonics. Implicit in the variational principle is that of all the possible flow fields, the one that minimizes E is the one that is realized. Therefore, the E

for the realized flow, which contains toroidal motion, must be less than the E for purely divergent flow, which is one of the many other possible flow fields. Thus it is to be expected that viscous dissipation is reduced as toroidal motion is generated since the viscous dissipation has the same scaling as E (i.e., it has the same dimensions and is controlled by the same properties such as η and $\dot{e}^2$).

Conclusion

The manner in which toroidal motion arises and interacts with the nonlinear plate-mantle rheology to reduce viscous dissipation provides some important guiding principles for how and why strike-slip motion should form in a convecting medium. Viscous drag acts to dissipate the convective energy used to transport mass along the surface from a source (e.g., a ridge) to a sink (e.g., a subduction zone). This dissipation is reduced, however, by focussing the deformation between the source and sink into narrow, essentially lubricated tracks or slip zones [see also Froidevaux, 1973]. Even though these zones have high $\dot{e}^2$, their viscosity η is small and they do not assume much area, thus making only a small contribution to the total dissipation. The broad regions outside these deformation zones do not contribute much to the net dissipation, either; although their area and viscosity may be large, they are nearly undeformed and thus $\dot{e}^2 \approx 0$. In contrast, if deformation were spread out, then both $\dot{e}^2$ and η would be significant in magnitude over a large area, causing a large net dissipation. It is no trivial clue that the self-lubricating rheology ($n = -1$), which yields the most plate-like focussing of toroidal flow and viscosity minima into narrow zones [Bercovici, 1993, 1995], yields the greatest reduction in dissipation. These results also suggest that convection models which permit viscous heating and temperature-dependent viscosity may induce greater and more focussed toroidal motion as the flow field attempts to reduce the net amount of viscous dissipation [e.g., see Balachandar et al., 1995]. It appears that strike-slip shear (i.e., toroidal motion) and thus plate-like flows themselves are generated to minimize the dissipation of poloidal motion and thus enhance the thermodynamic efficiency of the convective engine.

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