Rheology of the Earth's mantle: A historical review

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ABSTRACT

Historical development of our understanding of rheological properties of the Earth’s mantle is reviewed. Rheological properties of the Earth’s mantle control most of the important geological processes such as the style of mantle convection (e.g., stagnant lid versus plate tectonics) and the nature of thermal evolution. However, inferring the rheological properties of the Earth’s mantle is challenging because of the presence of multiple mechanisms of deformation that have different dependence on time-scale (strain-rate), stress levels and other parameters. Through the integration of a broad range of observations including the elastic stiffness from tidal deformation and the viscosity from the concept of isostasy, a gross picture of rheological stratification of the Earth’s mantle (a strong lithosphere, a weak asthenosphere and a strong layer below) was proposed in the mid-19th century. However, not only the physical basis for such a model was weak due to the lack of proper understanding of some materials science issues such as the interplay between elastic and viscous deformation but also the lack of understanding of temperature–depth relation associated with convection prevented our understanding of the rheological structure of the Earth’s interior. Major progress occurred in the first half of the 20th century in our understanding of the atomicistic mechanisms of plastic deformation in solids, and much of the theoretical framework on the plastic deformation of solids was established by late 1960s. Those developments provided a basis for scaling analyses that are critical to the applications of laboratory results to the Earth’s interior. Major progress in laboratory studies on rheological properties occurred in the mid-1960s to the early 1970s in Griggs’ lab using a new type of solid-medium high-pressure deformation apparatus to pressure ~ 2 GPa and temperature ~ 1600 K. The basic concepts such as the water weakening, non-linear rheology and deformation-induced lattice-preferred orientation were identified by their studies. However, large uncertainties in the stress measurements with this type of apparatus were recognized in the late 1970s and high-resolution experimental studies using synthetic samples were initiated in Paterson’s lab in the mid-1980s using a high-resolution gas-medium deformation apparatus. However, experimental studies with such a gas-medium deformation apparatus can be conducted only at low pressures (~ 0.5 GPa) and it is difficult to apply these low-pressure data to the Earth’s interior deeper than ~ 20 km. New experimental techniques to study rheological properties in Earth’s deep interior have been developed during the last a few years. These techniques allow us to quantitatively study the rheological properties of Earth materials down to the lower mantle condition (~ 24 GPa, ~ 2000 K). Some geodynamic issues related to rheological properties are discussed including (i) the strength of the lithosphere and the origin of plate tectonics, (ii) the origin of the asthenosphere, and (iii) the deep mantle rheology and its influence on thermal history of Earth. Existing models on these topics are reviewed and new alternative models are discussed.

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1. Introduction

In his classic articles “History and Current Status of Earth Science” published in 1965–1966 (reproduced as Miyashiro, 2009), Miyashiro emphasized the importance of understanding the historical development of geological sciences. After the in-depth analyses of the logical structure of geological sciences, he discussed that because geological science has intricate, multifaceted nature compared to physics, understanding the history of geological science is critical in order to understand various controversies or models in the proper context. In fact, in his textbook (Miyashiro, 1965, see also Miyashiro, 1973; Kushiro, 2010 —this issue), Miyashiro described the historical development of study of metamorphic rocks and metamorphism. This point was further expanded in his later book (Miyashiro, 1998) where he analyzed the differences in the logical structure between physics and geological sciences taking examples such as Bowen’s model of origin of volcanic rocks.

In this article, I present a historical review of studies of rheological properties of Earth’s mantle (and crust) following the spirit of Miyashiro. The intention is to identify some of the intriguing but potentially confusing issues in this area through the analysis of the history of this area. Therefore, those topics that have important relevance to some of the currently controversial issues were chosen.
Importance of understanding rheological properties of the mantle is obvious. Mantle convection controls most of the geological processes in terrestrial planets and the rheological properties of the mantle have the first-order influence on the nature of mantle convection. Some of the fundamental questions of Earth science such as “why does plate tectonics occur on Earth but not on other planets?” “how do the materials inside of Earth circulate and how are they mixed to control the evolution of this planet?” or “how has Earth evolved thermally?” can be addressed only when we understand the rheological properties of the Earth’s mantle. However, the studies of rheological properties of the Earth’s mantle are challenging and there have been many controversies in this area. Understanding the history of development in this topic will help appreciate the nature of controversies in this important area of research. Indeed, as discussed in this paper, the historical development of research on this area is complicated and interesting. When rheological properties are studied only from the material science point of view, the approach is straightforward and not much different from typical physical sciences. However, when studies of rheological properties of mantle are conducted to solve some geological problems, the multifaceted aspect of this area becomes immediately clear. Not only does one need to integrate a broad range of observations to come up with plausible models of rheological properties and related geological processes, but also considerations of geological problems often lead one to investigate deep into the materials science basis of deformation of materials. Some examples of the multifaceted nature of this area of science will be described.

This paper addresses the following topics:
• Mechanical properties of solids
• A brief history of studies of the mechanical properties of Earth
• Mineral and rock physics studies on rheological properties
• Mechanical properties of Earth’s interior
• Mechanical properties and mantle convection
• Concluding remarks

2. Mechanical properties of solids

2.1. Time-dependent deformation of solids

When stress is applied, materials will be deformed. Deformation can be classified into elastic and non-elastic. When a small stress is applied for a short time, then a material will be deformed instantaneously, and when the stress is removed, the material goes back to the initial state. This instantaneous and recoverable deformation is called elastic deformation. Material properties responsible for elastic deformation are characterized by elastic constants. In contrast, when a large stress is applied or a small stress is applied for a long time, then deformation occurs gradually and, in most cases, after the removal of the stress, the material does not revert to the initial state. Such time-dependent and often non-recoverable deformation is called non-elastic deformation. A typical non-elastic deformation is viscous deformation that is characterized by viscosity. The essence of elastic and viscous deformation was known from a study of Hooke in the 17th century (see Timoshenko, 1953). However, the first paper to discuss the operation of two modes of deformation in solids was by Kelvin (Thomson, 1865) where he demonstrated finite energy loss by the observation of amplitude decay of vibrating metals (Al, Cu, Zn). How does elastic and viscous deformation operate together in a material? There are two fundamentally different models of mechanical properties of matter involving both elastic and viscous component: the Maxwell and the Kelvin–Voigt models (Fig. 1).

\[ \tau_M = \frac{\eta}{\mu} \]  

(1)

where \( \eta \) is viscosity and \( \mu \) is shear modulus. For the Maxwell model, a material will behave like a viscous material at a long time-scale \( (t \gg \tau_M) \). In contrast, for the Kelvin–Voigt model, a material will behave like an elastic solid for the time-scale larger than \( \tau_M \). The Burgers model is a combination of the Maxwell and the Kelvin–Voigt model. The Burgers model explains (i) the instantaneous elastic response, (ii) long-term viscous response, and (iii) the intermediate time anelastic response of a material.

Fig. 1. (a) Maxwell, (b) Kelvin–Voigt model and (c) Burgers model of visco-elastic or anelastic behavior. For the Maxwell model, a material will behave like an elastic solid if the time-scale of deformation is much shorter than the Maxwell time, \( \tau_M = \frac{\eta}{\mu} \). In contrast, for the Kelvin–Voigt model, a material will behave like an elastic solid for the time-scale larger than \( \tau_M \).
viscous flow to a first-order approximation (note that at the time of Maxwell and Kelvin, the concept of thermally activated motion of atoms was not understood in the community). However, at a smaller time-scale, the motion of “defects” often generates some back-stress that will terminate defect motion. Such a back-stress is removed at a longer time-scale. Consequently, a more comprehensive model to describe the time-dependent deformation of a solid is the combination of the Kelvin–Voigt model and the Maxwell model called a Burgers model (Fig. 1c). The transition from Kelvin–Voigt to Maxwell model behavior described in the Burgers model corresponds to the change from transient to steady-state creep for a constant stress. Experimental evidence for such a transition was reported by Lakki et al. (1998). The usefulness of the Burgers model was discussed by Yuen et al. (1986).

The simplest version of the Burgers model, however, does not fully describe the time-dependent deformation of Earth, particularly seismic wave attenuation. Non-elastic deformation responsible for seismic wave attenuation is presented by the Kelvin–Voigt element in the Burgers model, but with only one Kelvin–Voigt element, one expects strong frequency dependence of seismic wave attenuation, viz., \( Q^{-1} \propto \omega^{-1} \), where \( Q^{-1} \) is the fraction of seismic wave energy dissipated as heat, \( \omega \) is frequency and \( T \) is the relaxation time. The observed frequency dependence of \( Q \) is more modest, \( Q^{-1} \propto \omega^{-\alpha} \), indicating that there is a wide distribution of characteristic times (Anderson and Given, 1982). Note, however, that the lower cut-off frequency will not exist due to the gradual transition to the viscous behavior at lower frequencies (Fig. 2). This point is important when one connects seismic wave attenuation to long-term viscosity (e.g., Karato, 1998a) and will be discussed later in relation to the Jeffreys’ argument against mantle convection.

\[ \dot{\epsilon} = A \left( \left[ 1 + \beta T^\gamma \right] \exp(kT) - 1 \right) \]  

where \( \dot{\epsilon} \) is strain, \( t \) is time, \( A, \beta \) and \( k \) are constants. However, his paper presents only a phenomenological description and there is no discussion on the microscopic mechanisms of deformation. The easiness of plastic deformation of metals (even at low temperatures) was explained by the concept of crystal dislocations by three papers published independently in 1934 (Brown, 1934; Polanyi, 1934; Taylor, 1934). A few years later, Peierls (1940) developed a theory for the stress necessary to move a dislocation at \( T = 0 \) K (this stress is called Peierls stress). This theory is refined by (Foreman et al., 1951) who considered the influence of chemical bonding (shape of interatomic potential) on the Peierls stress. The results of these two studies explain why plastic deformation is difficult in silicate minerals (at low temperatures) in which chemical bonding is strong (e.g., covalent bonding) and unit cell dimension is large.

At high temperature, plastic deformation occurs easily in many materials including silicate minerals. However, those models of dislocations cited above do not explain the importance of temperature on plastic deformation. By mid-1950s, experimental observations were obtained on high-temperature creep of metals by dislocation motion, in which a similarity in temperature dependence of creep rate and that of diffusion was noted. On the basis of these observations, Weertman (1955) proposed a model of high-temperature dislocation creep involving diffusion-assisted motion of dislocations (see also Weertman, 1968). When plastic deformation occurs by the motion of dislocations, strain-rate increases non-linearly with stress (and strain-rate is insensitive to grain-size).

Plastic deformation may also occur by the motion of atoms by diffusion. Nabarro (1948) proposed a model of high-temperature creep of polycrystalline solids by diffusional mass transport. This model leads to a linear relationship between stress and strain-rate similar to a typical viscous fluid. The viscosity corresponding to this model is proportional to some power of grain-size: smaller the grain-size easier is deformation. In his first paper on diffusion creep, Nabarro already pointed out the possibility of diffusion creep in Earth. Nabarro’s model was refined by Coble (1963) who included the influence of diffusive mass transport along grain-boundaries. Some modifications to large-strain diffusion creep were made by Ashby and Verrall (1973) to include the influence of grain switching processes.

Regarding high-temperature creep, what temperature is the high-temperature? \( T = 250 \) K is high temperature for ice but low temperature for olivine. To help understand this point, Sherby and Simmons (1961) introduced the concept of homologous temperature scaling in diffusion, i.e., \( D = D_0 \exp \left( -H^* / R \right) = D_0 \exp \left( -H^* / R \right) \) (\( D \) is diffusion coefficient, \( D_0 \) is a pre-exponential factor, \( H^* \) is the activation enthalpy of diffusion, \( \beta \) is a non-dimensional factor, \( T_0 \) is melting temperature, \( P \) is pressure, \( T \) is temperature, \( R \) is the gas constant). In this model, the
rate of diffusion (and hence diffusion-controlled creep rate) depends on temperature through $T/T_m$ and this normalized temperature is often referred to as homologous temperature. Sherby et al. (1970) extended this model to discuss the pressure effects by using $T_m(P)$ in the above equation. In parallel, Keyes (1958, 1960, 1963) proposed an alternative model to calculate the pressure dependence of diffusion from the pressure dependence of elastic moduli. These models have been used to estimate the pressure dependence of high-temperature creep with the assumption that high-temperature creep is controlled by diffusion (e.g., Sammis et al., 1977, 1981; Weertman, 1970).

The basic models of plastic deformation of materials at high temperature were developed in the first half of the 20th century, and these models were not available for the first generation of Earth scientists who speculated on mantle convection (Griggs, 1939a; Holmes, 1933). However, when the model of plate tectonics was developed (at around 1968), the basic concepts of high-temperature plastic flow were well established. McKenzie, one of the founders of theory of plate tectonics, for example, reviewed the basic knowledge of high-temperature creep available at that time (McKenzie, 1968).

Some materials scientists applied these models to discuss the rheological properties of Earth’s mantle (Stocker and Ashby, 1973; Weertman, 1970; Weertman and Weertman, 1975). An important addition was made by Goetze and Evans (Evans and Goetze, 1979; Goetze, 1978; Goetze and Evans, 1979) who noted the importance of high-stress, low-temperature deformation mechanism known as the Peierls mechanism for deformation of lithosphere using the formulation developed by Kocks et al. (1975).

The materials science studies of high-temperature creep summarized above suggest that important mechanisms of deformation in Earth are (i) diffusion creep, (ii) power-law dislocation creep and (iii) the Peierls mechanism. In all cases, deformation occurs by the thermally activated motion of defects and a generic form of flow law equation is

$$\dot{\varepsilon} = A \frac{\sigma}{n} \left(\frac{b}{a}\right)^m \exp \left(-\frac{E^* + PV^*}{RT}\right) = A \left(\frac{\sigma}{n}\right)^n \left(\frac{b}{a}\right)^m \exp \left(-\frac{E^* + PV^*}{RT}\right)$$

(3)

where $\dot{\varepsilon}$ is strain-rate, $\sigma$ is (deviatoric) stress, $A$ is a parameter with the dimension of frequency, $\mu$ is shear modulus, $b$ is the length of the Burgers vector, $d$ is grain-size, $n$ and $m$ are non-dimensional parameters, $E^*$ is activation energy, $V^*$ is activation volume, $\beta$ is a non-dimensional factor, $T_m$ is melting temperature, $P$ is pressure, $T$ is temperature, and $R$ is the gas constant (for the Peierls mechanism, the activation enthalpy depends on stress, but for simplicity, this term is not shown in Eq. (3)). The parameters in this flow law are different among the different mechanisms of deformation, and consequently, different mechanisms dominate under different conditions. A deformation mechanism map is a convenient way to visualize this and is a useful tool to infer the dominant mechanisms of deformation under a variety of conditions (Ashby, 1972). Stocker and Ashby (1973), for example, reviewed the then available experimental data to come up with a deformation mechanism map for olivine.

After the model of plate tectonics was accepted in the Earth science community in late 1960s, many Earth scientists started serious studies of rheological properties of mantle materials to explain plate tectonics and underlying processes such as mantle convection. The first extensive experimental studies on rock deformation were initiated in Griggs’ laboratory at UCLA in 1960s to early 1970s (those studies are summarized in Heard et al. (1972)). In the following years, various issues on plastic properties unique to Earth science have been recognized. These include (i) the influence of pressure, (ii) the influence of water, (iii) the influence of partial melting, (iv) the influence of deformation geometry and (v) the influence of phase transformations. I will review these relatively recent developments in a later section. In order to help understanding the link between the developments in materials science and geological sciences, I have prepared a table to summarize important developments in these two areas (Table 1).

3. A brief history of studies of the mechanical properties of Earth

3.1. Newton and the equilibrium shape of Earth

The idea of the hot and fluid-like Earth’s interior goes back to ancient Greek philosopher Empedocles (the 5th century BC). However, the first quantitative study of Earth’s interior was made by Newton. After describing his theory of gravity and mechanics in Part I and II of Principia (Newton, 1687), he provided a brief account of applications of his theory to the real world (mostly the shape of Earth and the Moon and their gravitational interactions including tides) in Part III. He calculated the flattening of rotating Earth assuming that the mass in Earth is distributed homogeneously (constant density) and that the hydrostatic equilibrium is established in Earth, i.e., assuming Earth behaves like a fluid. He obtained a value of $f = \frac{a}{2a-c} \approx 230$ (c: the equatorial radius, a: the polar radius) based on the measured values of gravity and the rate of rotation (centrifugal force). This value is close to the currently accepted value ($f \approx 298$) (the difference is mostly due to the depth variation of density in real Earth: Huygens calculated the factor $f$ assuming that the mass of Earth is concentrated in the center and obtained $f \approx 578$ (Huygens, 1690) (see also Mignard, 1987)). Note that if Earth behaved like elastic solid with plausible elastic constant ($\sim 200–300$ GPa), then the flattening would be much smaller (for a rigid model, f would be infinite). Given nearly spherical shape of planets, the assumption of fluid-like behavior of Earth’s interior (at static equilibrium) is an obvious choice from the continuum mechanics point of view. However, from the microscopic materials science point of view, the fluid-like behavior of deep Earth is not obvious. Newton did not give any explanation for the fluid-like behavior of Earth’s interior, but rather discussed such a model after discussing the shape of Jupiter. It appears that the deep interiors of all planetary bodies (including the Moon, see Proposition 38, Problem 19 of Principia) were considered to be liquid at these times. Given the present knowledge, we can translate the fluid-like behavior of Earth into the effective viscosity being less than $\tau_a/\mu$ ($\approx 10^{28}$ Pa s; $\tau$: age of Earth, $\mu$: elastic modulus of Earth) using the Maxwell model, but at Newton’s time, there was no estimate of the age of Earth nor of the elastic modulus of Earth’s mantle.

3.2. Isostasy

The idea of a soft interior of Earth obtained another support when the concept of isostasy was discovered. The basic observation that motivated the concept of isostasy was the gravity measurements in Peru by a French group (led by Bouguer in 1735 and 1745) and in India by a British group (led by Everest in 1847). The gravity measurements in these two expeditions showed that the gravitational attraction by the mountain belts in these regions (Andes and Himalaya) is much weaker than expected from their topography, if the mass distribution below the mountains is the same as the mass distribution outside of the mountain belts. Consequently, these observations were interpreted by the presence of low density materials below the mountains floating in a denser materials that behaves like a fluid (Airy, 1855; Pratt, 1855) (for a detailed account of history of the concept of isostasy, see Daly, 1940; Heiskanen and Vening Meinesz, 1958). Daly (1940) presented a quite modern view of regional isostatic compensation using a model of lithosphere–asthenosphere structure. Essentially, these observations imply that the deviatoric stress caused by the density heterogeneity is relaxed at a certain depth (in the asthenosphere) in the relevant geological time-scales. Given a plausible geological time-scale of $\sim 10$ Myrs and the deviatoric stress of $\sim 10$ MPa, one would get a viscosity of $\sim 10^{22}$ Pa s or less.
3.3. Kelvin and the mechanical stratification of Earth

The idea that the deep Earth is fluid was shattered by the finding by Kelvin (Fig. 3) (Thomson, 1863a) (a similar results was obtained using a rigidity (Dziewonski and Anderson, 1981). This must have been a surprise for him because almost all scientists where the value close to the current estimate of mantle rigidity (Dziewonski and Anderson, 1981). Kelvin recognized that the model of fluid-like deep Earth does not explain the observed tidal deformation. He found that it is necessary to assume that Earth behaves like an elastic material and considered the force balance due to gravitational force and elastic restoring force. In this model, tidal distortion depends on a non-dimensional parameter, where is rigidity, g is acceleration due to gravity, p is density and r is the radius of Earth. By comparing the observed tidal deformation with this model, he concluded that Earth has a rigidity that is several times higher than that of steel, the value close to the current estimate of mantle rigidity (Dziewonski and Anderson, 1981). This must have been a surprise for him because almost all scientific publications at that time (see Airy, 1855; Pratt, 1855 and also the model by Newton) assumed fluid-like mechanical properties of the deep mantle of Earth. In fact, in another paper published in the same year (Thomson, 1863b) where Kelvin calculated the age of Earth, he considered that the Earth’s interior was initially molten (citing the argument by Leibnitz), and his model age of Earth is the time necessary to cool initially molten Earth to the current thermal state by thermal conduction. However, he recognized that deep Earth is stiff, and consequently his paper on the age of Earth presents a convoluted model of Earth made of a thin crust (a hard layer) above a soft layer and the deep stiff region (Fig. 4). Kelvin argued that the freezing point of material will increase with pressure higher than that of temperature in Earth and justified such a model of mechanical properties of Earth. But, Kelvin did not have an idea of thermal boundary layer (i.e., inflection of geotherm from conductive geotherm to adiabatic geotherm) and therefore he could not explain why both near surface and deep layer are solid and a layer in between them is liquid.

As will be discussed in the next section, although such a model of rheological stratification is consistent with the current view of mechanical properties of Earth’s mantle (if a liquid layer is replaced with a soft solid layer), Kelvin’s arguments for such a model is not always valid and is highly confusing. Interestingly, Kelvin did not use Kelvin (~Voigt) model of anelasticity to come up with a rheological model of Earth when he used inferences of both elastic and viscous behavior. If Kelvin had used the Kelvin model of anelasticity, he should have concluded that the deep mantle should have a viscosity less than ~10^{18} Pa s (from the time-scale of ~3 x 10^{8} s, and the rigidity

Table 1
A brief history of study of deformation in materials and geological science.

<table>
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<th>Geological science</th>
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<td>2009</td>
<td>A modified theory of lattice strain</td>
</tr>
</tbody>
</table>

Notes to Table 1:

BOLD terms are important developments in Earth science.

Italics are important developments in experimental techniques of deformation.

1 : see (Timoshenko, 1953).
2 : (Newton, 1687).
3 : (Newton, 1865).
4 : (Maxwell, 1867).
5 : (Orowan, 1934; Polanyi, 1934; Taylor, 1934).
6 : (Peierls, 1940).
7 : (Nabarro, 1948).
8 : (Weertman, 1955).
9 : (Sherby et al., 1970).
12 : (Ashby and Brown, 1982).
13 : (Molinari et al., 1987).
14 : (Singh, 1993).
15 : (Weidner, 1998).
16 : (Karato, 2009).
17 : (Aiyer, 1855; Pratt, 1855).
18 : (Thomson, 1863a).
19 : (Ewing, 1880; Milne, 1880).
20 : (Adams and Nicolson, 1900).
21 : (Wegener, 1912a,b,c).
22 : (Gutenberg, 1926).
23 : (Holmes, 1931, 1933).
24 : (Haskell, 1935a,b).
25 : (Jeffreys and Bullen, 1940).
26 : (Griggs and Blacic, 1965).
27 : (Turcotte and Oxburgh, 1967).
29 : (Raith et al., 1969).
31 : (Paterson, 1970).
32 : (Blacic, 1972).
33 : (Dziewonski, 1984; Woodhouse and Dziewonski, 1984).
34 : (Hager, 1984).
35 : (Christensen and Yuen, 1984).
36 : (Karato, 1986) see also (Hirth and Kohlstedt, 1996).
37 : (Karato et al., 1986).
38 : (Fuku et al., 1992).
39 : (Jackson et al., 1992).
40 : (Zhang and Karato, 1995).
41 : (Solomatov and Moresi, 1996, 1997).
42 : (Paterson and Olggaard, 2000).
43 : (Jung and Karato, 2001).
44 : (Wang et al., 2003; Yamazaki and Karato, 2001a).
of $\sim 3 \times 10^{11}$ Pa), a conclusion opposite to what he suggested (Thomson, 1863b). If instead one used the Maxwell model of viscoelasticity, then one would have concluded that the deep mantle should have viscosity higher than $\sim 10^{18}$ Pa s. This lower limit of deep mantle viscosity is not higher than the viscosity of the shallow mantle inferred from isostasy, and does not lead to the rheological stratification suggested by Kelvin. The conclusion here is that Kelvin’s argument for the mechanical stratification (Thomson, 1863b) has no strong physical basis, if not incorrect.

3.4. Seismology and the mechanical properties of Earth

The modern seismology started when Milne and Ewing invented seismometers and used them in the late 19th century (Ewing, 1880; Milne, 1880). A major progress toward the mechanical properties of Earth’s interior was made by the advancement of seismology in early 20th century using newly developed seismometers together with the mathematical theory of elastic wave propagation (Herglotz, 1907; Wiechert, 1910). By seismology, one can investigate the mechanical properties of Earth at the time-scale of $\sim 1$ to $10^5$ s where the mechanical response is mostly elastic. The first studies of Earth structure based on seismological observations were published in 1930s by Jeffreys and Bullen (1940). These studies showed that Earth has a layered structure made of the crust, mantle and core, and the mantle on average has a large stiffness (large elastic constants) confirming Kelvin’s estimate of stiffness from tidal deformation. In more detail, using the travel time versus distance curves, Gutenberg (1926) discovered a layer of low seismic wave velocities just beneath a high velocity lid (the lithosphere). This layer corresponds to a weak layer inferred from the observations of isostasy, a layer called asthenosphere (Barrell, 1914). Gutenberg himself provided several explanations for the low-velocity zone including the presence of partial melt and the presence of glass-like materials. The origin of this weak layer is still controversial, and I will provide a detailed discussion on this issue later.

Seismic wave velocities are related to elastic properties and density and not much is learned about rheological properties from seismic wave velocities. However, from seismological records, one can also determine the magnitude of energy loss during seismic wave propagation. The energy loss is usually measured by a parameter, Q. Energy loss occurs via some viscous properties of solids, as first discussed by Kelvin (Thomson, 1865), so there is some link between Q and viscosity. However, the link between seismic wave attenuation and viscosity strongly depends on the mechanisms of by which elastic and viscous deformation occurs in a solid. If one uses the Kelvin–Voigt model, then $Q^{-1} \propto \eta$, whereas if one uses the Maxwell model, $Q^{-1} \propto \eta^{1/2}$. The correlation between attenuation and viscosity is opposite between the two: for Kelvin–Voigt model, high attenuation corresponds to high viscosity, whereas for Maxwell model, high attenuation corresponds to low viscosity. If one chooses a model of frequency-independent Q as often assumed in the seismological literatures (e.g., (Anderson et al., 1977)), then Q is insensitive to temperature and pressure, and the link between Q and viscosity is weak. Assuming the Maxwell model, (Anderson, 1966; Orowan, 1967) discussed the link between Q and viscosity. However, studies conducted later years showed the power-law behavior

$$Q^{-1} \propto \eta^{-\alpha}$$

with $\alpha \approx 0.3$ both from geophysical observations (Anderson and Minster, 1979) and from laboratory studies (e.g., Jackson, 2007). Knowing that Q is a non-dimensional quantity and therefore Q is a function of $\eta \tau$, this implies that

$$Q^{-1} \propto \tau^{-\alpha} \propto A_Q \exp \left( \frac{-\alpha H}{RT} \right)$$

Long-term viscosity depends on temperature and as other parameters as (from Eq. (3)

$$\eta = \frac{\sigma}{\tau} = A_\eta^{-1} \exp \left( \frac{H}{RT} \right).$$

Therefore there is a close relation between seismic attenuation and long-term viscosity when the relation (4) holds. For example, if both are dependent on temperature following relations (5) and (6), then

$$\frac{\eta_1}{\eta_2} = \left( \frac{Q_1}{Q_2} \right) \left( \frac{\tau_1}{\tau_2} \right)^{-\alpha}$$

with $\xi = \frac{H}{RT}$ (Karato, 2008). Because $\alpha \xi < 1$ (for the Maxwell model $\alpha \xi = 1$), a small variation in Q corresponds to a large variation in viscosity.
The contrast in $Q$ between the upper mantle and the lower mantle is about a factor of $\sim 4$ (Dziewonski and Anderson, 1981). If one uses the Maxwell model (Anderson, 1966), one would lead to a conclusion that there is only small viscosity contrast between the upper and the lower mantle (a factor of $\sim 4$), a conclusion that is consistent with that by (Peltier, 1989) but inconsistent with the current model of mantle viscosity (see the next section). In contrast, if one uses the relation (7) with $Q_0 \approx 0.3$, then one will find a viscosity contrast of $\sim 100$ that is consistent with the current best estimate of viscosity contrast.

Similar relationships exist for the influence of other factors such as water content and grain-size. Because seismic wave attenuation can be inferred with spatial resolution better than viscosity, a relation like (7) may be used to infer variation in viscosity.

3.5. Viscosity estimates from time-dependent deformation and/or gravity signals

Among various processes of time-dependent deformation, time-dependent vertical motion of crust after the last glaciation is the most frequently used observation that provides some constraints on the viscosity of the mantle. The melting of large ice sheets occurred rather quickly during 10,000 to 6000 years ago (e.g., Peltier, 1981). The melting resulted in re-distribution of load that caused slow flow of materials in Earth’s mantle that resulted in the time-dependent crustal motion. The vertical component of crustal motion can be measured as the relative sea-level change (Milne et al., 2001) also used GPS observations to add the horizontal component of crustal movement. The relative sea level (crustal uplift) follows approximately the relation,

$$z = z_0 \exp\left(-\frac{t}{\tau_0}\right)$$  
(8)

where $t$ is the height of crust relative to the sea level, $t$ is time, and $\tau_0$ is the characteristic time of crustal uplift. The time dependence of crustal deformation suggests that viscous deformation of materials is involved.

Such time-dependent deformation of the crust can be analyzed by using a model involving plastic deformation in the mantle beneath. In most cases, one assumes a linear (Newtonian) viscosity and some viscosity–depth relation is assumed. In a simple case, where only one representative viscosity is used, then one can obtain a relationship between the relaxation time and mantle viscosity,

$$\tau_0 = \frac{\rho_0 g N}{\pi \eta}$$  
(9)

where $\eta$ is viscosity, $\rho$ is density, $g$ is the acceleration due to gravity, $\lambda$ is the horizontal length scale of ice-sheet loading, and $F(\lambda) = \pi^2/\eta$ is a non-dimensional parameter that depends on the nature of viscosity stratification. For example, for an infinite layer with a constant viscosity, $F(\lambda) = 4\pi$ but for a case of a thin layer (thickness $H$) of low viscosity ($H=\lambda$), $F(\lambda) = \frac{3}{32}\left(\frac{H}{\lambda}\right)^3$ (e.g., Schubert et al. (2001), Chapter 18 of Karato (2008)). If all parameters other than viscosity are known, then one can determine the viscosity from this type of observation.

Note that in this formulation, only one viscosity is used. In practice, even at one locality where the observation is made, viscosity should vary with depth in more general way. By comparing the relaxation times for various space-scales ($\lambda$), it is possible to obtain some constraints on the depth variation of viscosity (e.g., Anderson and O’Connell, 1967; Cathles, 1975; McConnell, 1968; Mitrovica and Peltier, 1993). Essentially, in this approach, one interprets the viscosity in Eq. (9) as $\eta(\lambda)$. These studies suggest the presence of a low viscosity layer at around $\sim 100$–300 km depth but the depth resolution is poor.

A detailed theoretical analysis of post-glacial rebound was first performed by Haskell (Fig. 5) (Haskell, 1935a,b, 1937) assuming a homogeneous viscosity model. The average viscosity of the mantle is well constrained by his analysis and is $\sim 3 \times 10^{21}$ Pa s. Walcott (1972) introduced a concept of hydro-isostasy that plays an important role in the crustal movement in regions far from the glaciated regions. The inclusion of the concept of hydro-isostasy allowed the analysis of global data set of post-glacial rebound. Peltier (1974) and Cathles (1975) developed a mathematically elegant formulation of this problem. However, these studies led to a conclusion that the viscosity of Earth’s mantle is nearly constant with depth. This model dominated till mid-1980s (e.g., Peltier, 1989). However such a constant viscosity model is physically unreasonable if one uses a plausible materials science-based model of plastic deformation (e.g., Karato, 1981). Some authors proposed microscopic models to explain a nearly constant viscosity (O’Connell, 1977; Poirier and Liebmann, 1984) but these models are physically unsound (see Chapter 10 of Karato, 2008). The solution of this “puzzle” was provided by two studies. On the one hand, in a series of papers, Nakada and Lambeck showed that the analysis of relative sea levels involving hydro-isostasy must include the careful analysis of the influence of coast-line geometry (Nakada, 1986; Nakada and Lambeck, 1987, 1989). Because the relative sea-level change is observed at the coast-line, and the load distribution along the coast depends critically on the geometry of the coast line, large corrections are needed to extract mantle viscosity from such data. The global analysis of such data by Peltier (1989) and Cathles (1975) did not include these corrections and the validity of their results showing nearly depth independent viscosity is questionable. Also Mitrovica and Peltier (1991) showed that the relative sea-level data after the last glaciation are insensitive to mantle viscosity below $\sim 1200$ km depth (see also Mitrovica, 1996). However, through a careful analysis of regional relative sea level data with particular attention to the influence of coast-line, Nakada and Lambeck (1989) showed that some constraints on lower mantle viscosity can be obtained and they showed that the lower mantle viscosity is $\sim 100$ times larger than upper mantle viscosity. On the other hand, later measurements of activation energy and volume of diffusion coefficients in perovskite and (Mg, Fe)O showed rather small values, that explain nearly constant viscosity in the lower mantle (Yamazaki and Karato, 2001b) (although there could be a viscosity contrast between the lower and the upper mantle).

Slow motion of materials caused by re-distribution of the surface load also results in changes in the moment of inertia. Some geodetic measurements can provide an estimate of the change in moment of inertia with time. These data are sensitive most to the viscosity of the deep mantle. Peltier (1985b) and Yuen et al. (1982) reported the results of such studies.

Fig. 5. Norman A. Haskell (1905–1970) (from Ben-Menahem, 1990). A theoretical geophysicist who made the first definitive analysis of post-glacial rebound to estimate mantle viscosity. Haskell is also known his studies on seismology.
The time-scale of deformation associated with post-glacial rebound is on the order of several thousands years, and the crustal motion is definitely time-dependent providing a clear evidence of viscous behavior of Earth’s mantle. Although, the validity of analyzing such a phenomenon by viscous flow model (in the mantle) is clear, it should also be noted that the strain magnitude involved in this phenomenon is extremely small. A rough estimate of strain involved in this process can be obtained from \( \varepsilon \sim \frac{1}{T} \), where \( h \) is the vertical movement of crust and \( L \) is the horizontal scale. With \( h \sim 100 \) m and \( L \sim 1000 \) km (in near glaciated regions), one has \( \varepsilon \approx 10^{-4} \) (in the far field, \( h \sim 1 \) m and \( L \sim 1000 \) km, so strain is \( 10^{-6} \)). This strain is about the same as the elastic strain \( \varepsilon = \frac{h}{2L} \approx 10^{-4} \) where \( h \) is the thickness of the ice sheet (\( \sim 10^2 \) m) and \( \mu \) is shear modulus. The strain magnitude associated with the post-glacial isostatic adjustment is comparable to the elastic strain. Consequently, it is not obvious if the viscosity inferred from the analysis of post-glacial isostatic adjustment is the same as long-term viscosity relevant to mantle convection (Karato, 1998b). For the discussions on the influence of transient creep see Peltier (1985a) and Yuen et al. (1986).

At about the same time, Hager (1984) and Richards and Hager (1984) made a breakthrough in the analysis of gravity data in terms of mantle viscosity. Essentially Hager and Richards solved a puzzle of observed (weak) positive (long wavelength) gravity anomalies near subduction zones (Kaula, 1972): if heavy slabs sink into the mantle with a constant viscosity, then sinking slab should drag the near surface layer down so that one should expect negative gravity anomalies in the subduction zones. In fact, Kaula (1972) suggested that “the downthrust slab could in part be supported by stiffer matter below the asthenosphere” (see also Morgan, 1965a,b). Hager (1984) and Richards and Hager (1984) analyzed the gravity signals from subduction zones where the influence of heavy slab and the boundary distortions was included. They showed that if the mantle viscosity is homogeneous, then slabs sink deep into the mantle without much resistance leading to a large depression of the surface, hence this results in negative gravity anomalies. In contrast, if the mantle viscosity has a sharp increase in the mid-mantle, then slab subduction is (partly) inhibited and the surface topography due to the drag by a slab will be less. In such a case, the gravity signal is weakly positive. Through such an analysis, they concluded that the mantle viscosity should increase sharply by a factor of 30–100 at the mid-mantle depth (they assumed that this happens at 670 km).

Essentially, in this approach, one uses the density anomalies in the mantle as a given data set, and calculates the gravity field (including the distortions of various density boundaries) and by comparison with observed gravity field, one selects acceptable rheological models. In this approach, a key parameter is the viscosity contrast at density boundaries that controls the gravity signal. The absolute values of viscosity do not have any effects on the gravity signal. In the original papers (Hager, 1984; Richards and Hager, 1984), they used the calculated density anomalies of subducted slabs from thermal models to infer the depth variation of viscosity. In many later papers, density distribution is inferred from seismic tomography using the velocity to density conversion factors (e.g., Hager et al., 1985; King, 1995). Forte and his co-workers combined the relative sea-level change data, gravity data (and plate motion data) to infer the rheological models of mantle (Forte and Mitrovica, 2001; Forte et al., 1994). A major limitation with this approach is the uncertainties in the inferred density distribution. Some issues on the density–velocity conversion for thermal origin of anomalies were discussed by Karato (1993). But such a mineral physics analysis also shows that the density to viscosity conversion factor becomes poorly defined in the deep mantle where chemical heterogeneity plays an important role. For instance, Forte and Mitrovica (2001) inferred a high viscosity hill in the middle lower mantle (\( \sim 1500 \) km depth). However, the velocity to density conversion factor becomes highly uncertain near this depth range because of the strong influence of chemical composition (see Chapter 20 of Karato, 2008)), and I consider that the presence of a viscosity hill is questionable. Fig. 6 summarizes one-dimensional viscosity–depth models of Earth’s mantle inferred from these geophysical studies.

It should be noted that all of these methods have limited spatial resolution. However, from the consideration of mineral physics and mantle dynamics, one expects a large spatial variation in viscosity that may play important roles in geodynamics. For example, below mid-ocean ridges, dehydration reactions likely produced a large spatial viscosity contrast (Hirth and Kohlstedt, 1996; Karato, 1986). Ito et al. (1999) explored the influence of spatial variation in viscosity below mid-ocean ridges on the flow pattern and melt segregation. Similarly, a large variation in viscosity is expected in subduction zones (e.g., Billen and Gurnis, 2001; Karato et al., 2001).

In most of these geophysical inferences of mantle viscosity linear Newtonian rheology is assumed. As discussed before, the linear Newtonian rheology is not commonly observed in laboratory studies of rock deformation. Non-linear rheology corresponding to power-law dislocation creep is the most commonly observed mechanism of deformation in laboratory studies. When rheology is non-linear, then the relaxation time depends on stress and hence should change with time and location. Post and Griggs (1973) and Karato and Wu (1993) addressed the issue of non-linear rheology in post-glacial rebound by investigating possible deviation from a simple relation such as (8). However, the possible deviation from (8) is subtle and no definitive conclusion was obtained by these studies.

4. Mineral and rock physics studies on rheological properties

4.1. Introduction

Griggs (Fig. 7) was the pioneer and the most important scientist in the area of rock deformation as applied to geological sciences. His research was motivated by his strong interests in the physical mechanisms of a broad range of geological processes including mountain building, mantle convection and deep earthquakes. He recognized that in order to understand the physical mechanisms of these geological processes it is critical to understand the mechanical properties of rocks, and furthermore he also recognized that the apparatus that were available off-the-shelf were not enough to conduct key experiments needed to solve these geological problems.

![Fig. 6](image-url)
Consequently he developed a range of new apparatus for rock deformation. The need for experimental studies on deformation of minerals and rocks is eloquently described by Griggs (1936): “We have many books filled with geological observations and speculations concerning the behavior of rocks under the conditions of high pressure and temperature which prevail in the outer shell of the earth. We observe their characteristics after they have been plunged deep into the earth, fractured, deformed plastically, metamorphosed, uplifted again, and exposed by erosion. As to the actual physical processes which cause these changes we are pretty much in the dark, partly because we cannot tell how many of the processes took place simultaneously, and partly because we can never be sure of the environment in which these changes took place—the conditions of temperature and pressure and the role of mineralizing solutions. The most promising method of increasing our knowledge of the physics and chemistry underlying these processes seems to be experimentation in the laboratory, under controlled conditions of pressure and temperature.”

It is true that one needs experimental studies to learn how minerals and rocks might be deformed in geological processes, and to understand how geological phenomena such as mountain building, mantle convection and earthquakes might occur. However, experimental studies of deformation of minerals and rocks are not straightforward for several reasons. First, there are many processes or mechanisms of deformation depending on the physical and chemical conditions of deformation. Therefore one needs to identify which mechanisms might be relevant to decide under what conditions one may conduct deformation experiments. Second, in many cases, deformation properties are time-dependent. In other words, the response of minerals or rocks to external deviatoric stress depends on the rate at which deformation occurs. And in many cases, the rate of deformation under geological conditions is much lower than typical rate of deformation that one can investigate in the laboratory. Consequently, one needs large extrapolation, and the evaluation of validity of such extrapolation is not straightforward. Third, in comparison to experimental studies of more static properties such as equation of state or elastic properties, quantitative studies of plastic deformation are more complicated because the rate of deformation needs to be controlled, and also because plastic properties are sensitive to subtle variation of some parameters such as water fugacity. In fact, Griggs (1936) complained: “Experimental observation of the deformation of rock under high confining pressure has been greatly neglected since the pioneering work by F.D. Adams. The bulk of the experimental work has been concerned with physico-chemical relations of mineral stability and with compressibility measurements. It seemed important to the writer to check Adams’ work and to carry the study of rock deformation to the higher values of confining pressures which are now available in the laboratory.” Such a situation has not been changed so much after ~70 years since Griggs noted. In our days, not only geological observations but also a range of geophysical observations such as seismological observations suggest importance of plastic deformation of rocks in many geological processes, yet the experimental studies on rock deformation were limited to low pressures because not much technical developments were made after Griggs and Paterson. Until very recently, high-resolution, quantitative long-term deformation experiments were limited to less than ~1 GPa, and there were very poor constraints on the plastic properties in Earth below ~20–30 km depth. Before discussing the issues of deformation apparatus, let me first review the scaling issues.

4.2. Scaling issues

One of the important questions in the laboratory studies of plastic deformation in Earth science is how to fill the gap between laboratory studies and actual deformation in terms of time-scale and space-scale. The sample size that one can study in a lab study is ~1–10 mm compared to ~10–1000 km (or more) in real Earth. Typical strain-rates in laboratory studies are $10^{-6}$–$10^{-3}$ s$^{-1}$ but the typical geological strain-rates are $10^{-12}$–$10^{-10}$ s$^{-1}$. Similar contrasts in space- and time-scales exist in the measurements of static properties such as elastic properties and equation of state. For instance, in the Brillouin scattering measurements of elastic properties, one uses the frequency of $10^4$ Hz compared to the seismic frequencies of $10^{-3}$ Hz. And the contrast in space-scale is similar to that for deformation studies. However, the differences in scales do not cause major problems in the study of static properties. In terms of time-scales, elastic properties are affected only modestly by frequencies (by anelasticity; Jackson, 2007), and the equation of state is not affected much by frequencies except for liquids (e.g., Dingwell and Webb, 1990). Also the issues of space-scale are minor in these properties. For example, differences in elastic properties at each grain scale and rock scale is minor (e.g., Watt et al., 1976). This is essentially due to the fact that the anisotropy and heterogeneity in static properties (such as elastic properties) are small in these properties. Elastic anisotropy and differences in elastic constants are on the order of ~10%, so averaging issues will cause the differences in calculated (or measured) elastic properties only a few % or less (only important except as a mixture of solids and liquids).

The situation is quite different for plastic deformation. The space-scale issues can be important for plastic properties. This is due to the fact that anisotropy and heterogeneity are large for plastic properties. Plastic anisotrophy of minerals can be large, exceeding a factor of ~100 in many cases (e.g., ice: Kocks and Canova, 1981, forsterite: Darot and Gueguen, 1981)). Heterogeneity in plastic properties can also be large. For example, when grain-size variation of a factor of ~10 occurs, this could cause the variation in effective viscosity on the order of ~$10^3$. Also the differences in effective viscosity among co-existing mineral can be more than one order of magnitude. For these reasons, scaling in terms of space can be an important issue.

Let us consider the issue of shear localization. Localization of deformation is frequently observed in nature (e.g., White et al., 1980). The causes of localization include grain-size reduction, localized deformation in a weaker phase and thermal runaway (see Chapter 16 of Karato, 2008 for more details). In these cases, shear zones with concentrated deformation will be developed. Those zones have some characteristic length scale (spacing of concentrated shear). If the sample size is smaller than the characteristic space-scale, then one would not observe shear localization in a small sample in the lab, although shear localization might occur at larger scale in Earth. (Holtzman et al., 2003b)
conducted an experimental study of deformation of partially molten peridotite where they controlled the characteristic length (this case the compaction length) to be smaller than the sample thickness by adding secondary minerals to reduce the permeability. In this way, they were able to investigate the nature of shear localization in the melt-rich layer in laboratory studies with a sample dimension of ~1 mm. A similar situation might exist in shear localization caused by grain-size reduction, but no detailed studies have been conducted on this topic in the laboratory. Post (1977) reported ductile faulting in dunite in his lab studies, but the mechanical data from his study were subjected to large uncertainties.

Time-scale issues in deformation studies are well known (e.g., Paterson, 1987), but let me summarize these issues because there is still a lack of understanding of seriousness of extrapolation in time-scale (and in temperature). There are many papers on high-pressure deformation in which the results from experiments with unconstrained rate of deformation at room temperature were used directly to discuss deformation in Earth’s deep interior such as the lower mantle (e.g., Meade and Jeanloz, 1990; Merk et al., 2007). In these papers, possible differences in deformation mechanisms between those in experiments and those in Earth were not discussed, and the applicability of these results to deformation in Earth is highly questionable.

The essence of time-scale issue is that because no deformation experiments can be conducted at geological time-scale, large extrapolation in time-scale (strain-rate) is needed in applying the experimental results to deformation in Earth’s interior. Extrapolation is made in such a way that one determines the relationship between strain-rates and other variables (stress, temperature, pressure, grain-size etc.) in the lab, and uses such a relation to calculate the effective viscosity at geological strain-rate. The important point is that the functional form of flow law depends on the mechanisms of deformation as discussed above, and that the mechanism of deformation changes with physical conditions including strain-rate. Therefore in order to justify such extrapolation, one must determine the mechanisms of deformation in the laboratory experiments and also infer the likely mechanisms of deformation in Earth.

To illustrate this point, let us examine a deformation mechanism map for olivine (Fig. 8). In this diagram, strain-rates corresponding to three important mechanisms of deformation (diffusion creep, power-law dislocation creep and the Peierls mechanism) are compared and assuming that these three mechanisms operate independently, the dominant mechanisms of deformation were identified for a given parameter space (in this figure I use stress and grain-size as variables for a fixed pressure, temperature and water fugacity). It is seen that under geological conditions (deep upper mantle), likely mechanism of deformation is either power-law dislocation creep or diffusion creep. In contrast, in typical laboratory deformation experiments, all three mechanisms could operate appreciably depending on the precise conditions of deformation such as grain-size and the stress level. Consequently, only some sub-sets of laboratory data correspond to deformation mechanisms that might operate in Earth’s deep interior. It is seen from this discussion that an extensive data set on flow laws of all potentially important mechanisms needed to be determined and a careful analysis of deformation mechanisms that may have operated in the lab studies and those in Earth needs to be conducted to justify the applications of laboratory data to deformation in the Earth’s interior.

Analysis of deformation mechanisms in laboratory experiments is straightforward (see for e.g., Karato, 2008; Poirier, 1985). One uses typical techniques of materials science of deformation: deformation microstructures and flow laws. For example, different mechanisms lead to different types of flow laws. Diffusion creep shows a linear, grain-size dependent rheology, \( \dot{\varepsilon} \propto \sigma^m \) with \( m = 2-3 \), whereas dislocation creep at modest stress results in non-linear relationship, \( \dot{\varepsilon} \propto \sigma^n \) with \( n = 3-5 \) and \( m = 0 \), whereas at high-stress when glide-controlled mechanism operate, \( \dot{\varepsilon} \propto \sigma^m \exp(-\frac{C_16}{RT}) \). Therefore by determining the flow law precisely, one obtains robust conclusion as to the operating mechanisms of deformation. Such an inference can also be strengthened by the study of deformation microstructures. In many cases, deformation microstructures are studied on samples “quenched” at the end of an experiment. If cooling is rapid enough (sometimes cooling under stress is preferred to better preserve the microstructure), microstructures can be frozen (quenched). Diffusion creep results in no or little internal strain gradient in each grain, whereas dislocation creep results in strain gradient that can be identified by some microscopic observations. Also, deformation by diffusion creep usually results in no or weak lattice-preferred orientation (LPO), whereas dislocation creep results in strong LPO. By combining these two types of observations, deformation mechanisms operating in laboratory experiments can be identified in most cases.

In contrast to laboratory studies, the inference of deformation mechanisms in the Earth’s interior can be made only indirectly. The form of flow law operating in nature is difficult to infer. Karato and Wu (1992) and Post and Griggs (1973) used the observed deviation of crustal uplift associated with post-glacial rebound to infer if non-linear viscosity might operate. However, the resolution of the observed data is not enough to obtain robust conclusions. More useful constraints on deformation mechanisms can be obtained from microstructural observations. One approach is to observe microstructures of naturally deformed rocks. Plastic deformation occurs in relatively deep regions of Earth (below the lower crust). Therefore one needs to have access to rocks coming from the deep interior of Earth. Rocks from deep Earth often show microstructural evidence of plastic deformation and from which one can infer the operating mechanism of deformation. However, there are two major issues in this inference. First, all of these rocks coming from deep interior of Earth have complicated history. The microstructures that we observe in these rocks record all the thermal and mechanical history and we need to untangle the history to identify the operating mechanisms in the deep interior of Earth. Second, among various deformation mechanisms, some are directly recorded in deformation microstructures but others are not. In the latter case, careful analysis is required to assess the likely mechanisms of deformation in Earth’s interior.

Let me first discuss the issue of untangling the history from the microstructures of naturally deformed rocks. All deep Earth rocks are on Earth’s surface as a result of some geologic processes. In some

Fig. 8. A deformation mechanism map of olivine (at \( P = 7 \) GPa and \( T = 1700 \) K) (after Karato, 2010).
cases, we have evidence that a rock came from the mantle. Usually, this is inferred from the chemical composition and/or crystal structure of minerals. These observations tell us the temperature and pressure conditions at which a rock finally established the chemical equilibrium. If one finds evidence of disequilibrium, then one can infer pressure–temperature history (P–T–t path) (e.g., Spear and Peacock, 1989). Reading the history of deformation is more complicated, because the processes by which deformation microstructures may be modified are not as well as those of chemical composition. However, based on experimental studies, we have some ideas as to the processes of generation and destruction of some microstructures including dislocations, grain-boundaries and lattice-preferred orientation. Briefly, lattice-preferred orientation is the most stable microstructure among them (e.g., Heilbronner and Tullis, 2002; Zhang and Karato, 1995), dislocation density is the most unstable microstructure (e.g., Durham et al., 1977; Karato and Ogawa, 1982) and grain-size is in between (e.g., Karato, 1989b; Karato et al., 1980).

Second, the above-mentioned microstructures are mostly related to deformation by dislocation creep. Deformation by dislocation creep results in clear microstructural signature such as lattice-preferred orientation. In contrast, deformation by diffusion creep results in almost no positive microstructural signature. Absence of lattice-preferred orientation despite strong evidence of large strain is strong evidence for diffusion creep or superplasticity (e.g., Behrmann and Mainprice, 1987). However, such a null-result can be obscured easily by subsequent deformation. If the transport processes of rocks involve higher stress than those associated with slow deformation in the ambient mantle, then high-stress event during transportation will erase any previous random LPO. The presence of four-grain junctions is a strong evidence for grain switching events that can be used to identify superplasticity (e.g., Goldsby and Kohlstedt, 2001; Karato et al., 1998a). Again, however, four-grain junction geometry is easy to be reset to more stable three-grain junction geometry and it will be difficult to observe them if a rock is subsequently annealed or deformed by dislocation creep. By combining various microstructures, one can reveal the deformation history, but the interpretation is usually not unique (e.g., Mercier, 1979; Skemer and Karato, 2008).

An alternative approach is remote-sensing approach using seismological observations. Seismic waves penetrate deep into Earth’s interior, and carry detailed information of elastic (and anelastic) properties of Earth. Particularly important in terms of deformation mechanisms is seismic anisotropy. In most cases, seismic anisotropy is caused by the lattice-preferred orientation of elastically anisotropic minerals (e.g., Chapter 21 of Karato, 2008). Therefore the presence of strong seismic anisotropy can be taken as evidence for deformation by dislocation creep, and conversely absence of anisotropy is evidence for diffusion creep (or superplasticity). Karato et al. (1995) showed that the strength of anisotropy in the main part of the lower mantle is less than ~1/100 of what one expects from the LPO of lower mantle minerals deformation by dislocation creep, and suggested that the majority of the lower mantle is deformed by diffusion creep (or superplasticity).

In the early studies such as Zharkov (1960) and Gordon (1965), it was assumed that deformation in the Earth’s mantle occurs by diffusion creep without strong reasons. Other limitations of this approach are (i) they assumed grain-size without much reasons (Zharkov: 3 × 10^{-5}–3 × 10^{-4} m, Gordon: 5 × 10^{-4} m), and (ii) they did not have experimental data on diffusion coefficients in minerals in the Earth’s interior under deep Earth conditions, and (iii) the assumed geotherms have large uncertainties. Therefore this type of approach has highly limited usefulness. For example, a typical grain-size of upper rocks is ~10^{-3}–10^{-2} m (e.g., Avé Lallemant et al., 1980) and with these values, the viscosity would be different from those calculated by the above authors by a factor of 10^2–10^6. In contrast, in a more recent approach by Yamazaki and Karato (2001b), they first used the observed lack of seismic anisotropy in the lower mantle as evidence for diffusion creep. Then they used the measured diffusion coefficients in the lower mantle minerals to evaluate a range of acceptable grain-size and geotherm. Furthermore, they used the experimental results on grain-growth kinetics in lower mantle materials under the lower mantle conditions (Yamazaki et al., 1996) to argue that grain-size does not change with location so much. Based on these arguments, they infer the plausible range of geotherm and grain-size that is consistent with inferred depth variation in viscosity.

In contrast to these studies, Weertman (1970) and Stocker and Ashby (1973), for example, argued that the power-law dislocation creep is the most important mechanism of deformation in Earth’s whole mantle and in the upper mantle respectively. However, these arguments were not based on reliable experimental results. For example, in 1970s, there was not data on diffusion or diffusion creep in mantle minerals. Combining the experimental data on deformation and diffusion with seismological observations on anisotropy is a more solid way to identify the dominant mechanism of deformation in Earth. Using this approach, Karato et al. (1995) presented a strong case for diffusion creep (or superplasticity) in the lower mantle, but with dislocation creep in some regions of D’ layer (Karato, 1998c) and in the shallow upper mantle (Karato, 1992; Karato and Wu, 1993). Deformation mechanisms in the transition zone are poorly constrained. If seismic anisotropy is strong as suggested by Trampert and van Heijst (2002), then dislocation creep is the dominant mechanism of deformation. However, the model by Karato et al. (2001) suggests that in some portions of slabs in the transition zone, deformation is likely by diffusion creep or superplasticity after the significant reduction in grain-size.

4.3. Development of high-pressure and temperature deformation apparatus

An experimental study of deformation of rocks or minerals involves two aspects. First, one needs to create appropriate temperature, pressure and other thermodynamic conditions (such as the fugacity of water), and applies deviatoric stress or strain in a controlled fashion. Second, in addition to the measurements or the calculation of temperature, pressure and chemical factors (e.g., water fugacity or water content), one needs to measure both strain and stress. In the early stage of rock deformation studies, it was realized that a piece of rock is deformed by fracture at low pressures, whereas the mode of deformation gradually changes to more distributed strain called ductile deformation. Consequently, when one wants to study plastic flow, one needs a confining pressure. It was also noted that temperature also helps ductile deformation as opposed to brittle fracture. Many efforts have been paid to understand this brittle–ductile transition (or brittle–plastic transition) (for a review see a book edited by Duba et al. 1990). In this paper, attention is focused on the experimental studies of plastic deformation.

A serious experimental study of rock deformation was initiated by Adams and Nicolson (1900) on marble in which jacketed samples of marble were deformed under confining pressure surrounded by solids. Griggs (1936) modified Adams–Nicolson’s apparatus to allow more accurate measurements. Interestingly Griggs originally designed a gas-medium apparatus (not a solid-medium “Griggs apparatus”) to improve the measurements of pressure, and to allow a specimen to react freely under stress. A further modifications to the apparatus were made by Griggs et al. (1960) using an internal furnace to increase the maximum pressure of operation. One limitation of these apparatus is that the stress was measured by an external load cell and therefore the errors caused by friction is large (e.g., Gleason and Tullis, 1993). Also they used a gas (CO_2) that freezes at low pressure, so that the maximum pressure was limited to ~0.5 GPa. (Griggs, 1967) further improved this type of deformation apparatus by introducing a solid as a pressure medium. Essentially, this apparatus (called the “Griggs apparatus”) is a modification to the piston-cylinder apparatus
to allow independent control of deviatoric stress (strain) (Fig. 9a). He was able to operate this apparatus to \( \sim 2 \) GPa and \( \sim 1500 \) K. Although the mechanical measurements particularly the stress measurements with this apparatus have large errors, most of the important concepts and topics in rock deformation studies were identified or discovered by Griggs’ group using this apparatus including the weakening effect of water on quartz and olivine and the development of lattice-preferred orientation by plastic deformation and its relation to seismic anisotropy.

Some modifications to a liquid (gas)-medium apparatus were made by Heard and Carter (1968) to operate it to \( \sim 1 \) GPa and \( \sim 1300 \) K using Ar as a confining medium. However, the major limitation is that the load measurements were made using a load cell that is partly located outside of the high-pressure chamber. Consequently, the stress measurements are affected by the pressure fluctuation and are not accurate. (Paterson, 1970) designed a new type of deformation apparatus in which the load cell is located completely inside of the high-pressure chamber (Fig. 9b). The problems with friction are

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Fig. 9. Diagrams of various apparatus for experimental studies of deformation of minerals and rocks. (a) a modified piston-cylinder apparatus (Griggs, 1967). (b) a gas-medium deformation apparatus with an internal load cell (Paterson, 1970). (c) a modified cubic apparatus for deformation studies under high pressures (D-DIA) (Wang et al., 2003). (d) a modified Drickamer apparatus for deformation studies (RDA) (Yamazaki and Karato, 2001).
completely eliminated that results in the large improvement to the stress measurements (see also Paterson, 1990). A number of modifications to the proto-type apparatus have been made to increase the maximum temperature of operation, better temperature distribution, and in early 1980s this apparatus started to be used routinely to characterize plastic flow of minerals and their aggregates (an independent control of pore pressure and the capability of torsion tests were added later). In order to control water content and grain-size that affect plastic deformation, Karato et al. (1986) used synthetic aggregates of olivine. The combination of high-resolution apparatus and the use of synthetic samples with well-controlled water content and grain-size allowed them to establish the water-weakening effects and the transition conditions from diffusion to dislocation creep in olivine. This approach was followed by Kohlstedt’s lab in their extensive studies on olivine and other minerals (e.g., Hirth and Kohlstedt, 1995a,b; Mei and Kohlstedt, 2000a,b). Because of the high-resolution mechanical measurements, and of a good control of key parameters such as water content and grain-size, results from these studies are most reliable and can be used to identify the mechanisms of deformation with the highest confidence.

However, there is a major limitation of this apparatus: it can be operated only at relatively low pressures (∼0.5 GPa) above which sealing of a moving piston becomes difficult. Also the maximum temperature of operation is rather limited (∼1600 K). In particular, the limitation of pressure is serious for two reasons. First, some minerals are stable only at high pressures. For example, orthopyroxene is stable only above ∼1 GPa at high temperatures. Wadsleyite is stable above ∼14 GPa. The plastic flow properties of these important minerals cannot be studied using the Paterson-type a gas-medium apparatus. Second, because the influence of confining pressure is small at low pressures, the pressure effects are very difficult to determine using this apparatus. Third, the influence of water is a non-monotonic function of pressure when water fugacity is controlled by changing the confining pressure. The relation between the creep strength and pressure under water-saturated conditions changes at ∼0.5 GPa where the behavior of water changes from nearly ideal gas to highly non-ideal gas. In short, although high-resolution mechanical data can be obtained using the gas-medium deformation apparatus such as the Paterson apparatus, the applicability of these results are limited to below ∼20 km depth. In order to understand the whole Earth dynamics, it is necessary to develop new apparatus by which quantitative studies of plastic deformation can be made at higher pressures.

Not much progress in the experimental technique of studying plastic properties was made from early 1970s to early 2000s except for the improvement to the stress measurements with the Griggs apparatus using a liquid-cell (Borch and Green, 1989; Green and Borch, 1987). The reason for this stagnant period is unknown but it is presumably due to the fact that “rock deformation” community was dominated by structural geologists and there were not many demands for high-pressure experimentation. Starting from mid-1980s, high-resolution tomographic images of whole mantle became available (e.g., Dziewonski, 1984; Dziewonski and Woodhouse, 1987; Fukao et al., 1992, 2009; Zhao, 2009), and furthermore anisotropic structures have been revealed at the global scale (e.g., Montagner and Tanimoto, 1990, 1991; Panning and Romanowicz, 2006; Raat et al., 1969; Tanimoto and Anderson, 1984). Those new observations motivated some of us to develop new types of deformation apparatus to explore the dynamics of Earth’s interior based on the experimental data on rheological properties (for a review see Karato and Weidner, 2008). Technical challenges in this development are (i) to design apparatus in which deviatoric stress (strain) can be generated in a controlled fashion and (ii) to develop a technique to measure both stress and strain under high-pressure and temperature conditions. A few apparatus have been designed, and I will mention two of them, D-DIA (deformation DIA, Wang et al., 2003; Fig. 9c) and RDA (rotational Drickamer apparatus, Nishihara et al., 2008; Yamazaki and Karato, 2001a) (Fig. 9d). In D-DIA, a sample is squeezed by six anvils along the diagonal directions (this is why it is called DIA apparatus). Among them two opposed anvils can be advanced independently from the other four that enables one to generate a deviatoric stress in the controlled manner at high pressures. However, the support for anvils is poor with this design, and consequently the maximum pressure of operation is limited. RDA is a modified Drickamer apparatus where one of the anvils is rotated relative to another. The advantages of this design are (i) the capability of high pressure and temperatures experiments due to a good support for anvils, and (ii) the simple shear geometry that allows large-strain deformation experiments that are essential to the study of microstructure evolution during deformation. Deformation experiments have been performed to ∼23 GPa and ∼2200 K (Hustoft et al., submitted for publication).

In addition to the apparatus developments, the developments of in-situ technique of stress and strain measurements are also critical. Since a sample is located in a small space under high-pressure experimentation, it is impossible to use conventional techniques of stress and strain measurements using a load cell and a displacement transducer respectively. In-situ stress–strain measurement techniques have been developed by Weidner’s group at Stony Brook University (Chen et al., 2004; Weidner, 1998; Weidner et al., 1998). Strain is measured by direct X-ray imaging and stress is calculated from the lattice distortion measured by X-ray diffraction. Although strain measurements by X-ray imaging are straightforward, stress estimated from X-ray diffraction is not. In the stress measurements from X-ray diffraction, one uses the dependence of lattice strain on the orientation of diffracting planes with respect to the macroscopic stress orientation. Singh (1993) developed a theory to calculate the macroscopic strain in a sample from X-ray diffraction measurements along various directions. This model was used in most of high-pressure studies of mechanical properties, but a major limitation of this theory was identified by Li et al. (2004), Weidner et al. (2004) and Chen et al. (2006) who showed that the actual experimental observations deviate far from Singh (1993)’s model prediction.

Karato (2009) developed a modified theory in which the role of plastic deformation by non-linear power-law creep is included. Combining the developments in these two areas, the conditions under which quantitative deformation experiments can be made have been expanded to much wider pressure and temperature conditions than ~10 years ago (Fig. 10). The main results of these developments include (i) the determination of creep strength of olivine to ~10 GPa and ~2000 K (Kawazoe et al., 2009), (ii) the flow law of serpentinite to 4 GPa and ~800 K (Hilaterial et al., 2007), and (iii) the quantitative results of deformation of transition zone minerals (Hustoft et al., submitted for publication; Kawazoe et al., In press).

Another important aspect of experimental studies of deformation is the extension of geometry of deformation into lower symmetry. In most of conventional deformation studies, a sample is squeezed uniaxially under a confining pressure. This is one of the co-axial deformation geometries in which there is no rigid body rotation (no vorticity). However, in many cases in real Earth, deformation geometry often involves rotational component such as simple shear. Rheological properties and deformation microstructures such as lattice-prefered orientation (LPO) are likely dependent on deformation geometry. For example, asymmetry of LPO with respect to the kinematic framework of deformation such as foliation and lineation is often used to infer the sense of shear (Bouchet et al., 1983). Torsion tests are used to investigate the influence of deformation geometry (e.g., Bybrick et al., 2001; Kamb, 1972; Paterson and Olgaard, 2000; Pieri et al., 2001), but also quasi-simple shear deformation experiments are often conducted using a tri-axial compressional testing apparatus using the saw-cut pistons (Jung and Karato, 2001; Schmid et al., 1987; Zhang and Karato, 1995). These low-symmetry
geometries are also suitable for attaining a large strain that is essential for the study of evolution of microstructures that is critical to the study of the structural geology of the mantle (Karato et al., 1998b). However, both of these techniques have limitations. First although there is no theoretical limit of strain in a torsion test, there is a strain (and hence) stress gradient in a sample that complicates the interpretation of the mechanical and microstructural observations (Paterson and Olgaard, 2000). Second, the magnitude of strain is limited in saw-cut quasi-simple shear deformation geometry, and deformation is only approximately simple shear and there is finite contribution from uni-axial compression. Contribution from this artifact often complicates the interpretation of deformation microstructures (Holtzman et al., 2003b; Karato et al., 2008).

Another application of these low-symmetry deformation experiments is to characterize plastic anisotropy. Plastic deformation is anisotropic in the same way as elastic deformation. This is obvious when one deforms a single crystal. A single crystal deforms along certain directions easier than others. Even in a polycrystalline sample, plastic anisotropy may be present. After deformation, a polycrystalline sample often develops lattice-preferred orientation that leads to plastic anisotropy. Plastic anisotropy might have some effects on mantle convection or other tectonic processes (e.g., Christensen, 1987; Honda, 1986; Lev and Hager, 2008; Saito and Abe, 1984), but not many studies have been made on this topic (e.g., Takeshita, 1989; Wendt et al., 1998).

4.4. Some specific problems on rheological properties unique to Earth science

In the section on mechanical properties of solids, a brief summary of the history of development of materials science of plastic deformation is attempted. The essence of the theory of high-temperature deformation was established in the mid-1960s. However, a few specific points must be discussed that are relevant to geological science. These are the influence of pressure, effects of partial melting and the effects of water. Goetze (Fig. 11) was a pioneer who made important contributions to all of these areas including some geological and geophysical applications of these results (Goetze, 1975, 1977; Goetze and Evans, 1979; Sung et al., 1977). Particularly notable are (i) the introduction of the concept of a strength profile (Goetze and Evans, 1979), (ii) the recognition of the importance of the Peierls mechanism in the deformation of cold slabs (Goetze and Evans, 1979), (iii) the outline of strategy for the study of the influence of partial melting and water on plastic flow (Goetze, 1977), and (iv) the pioneering work on ultrahigh-pressure deformation (Sung et al., 1977). For example, a review paper by Goetze (1977) formed the basis for the extensive studies in Kohlstedt’s lab on the influence of partial melting and the role of water on olivine deformation as discussed later.

4.4.1. Effects of pressure

It is often mentioned that plastic flow is insensitive to pressure but brittle fracture strength is sensitive to pressure. However, this statement is correct only at low pressures. Because the pressure effect on plastic flow is mainly through the influence of pressure on the activation enthalpy for deformation, its effect is small at low pressures, but becomes very large at high pressures. At the same time, this means that measuring the pressure effects at low pressures is difficult. Karato (2010) showed that although low-pressure experiments (P<0.5 GPa) using a gas-medium apparatus have higher resolution in mechanical measurements, the pressure range covered by this apparatus is too small to obtain any reliable estimate of pressure effects of deformation for typical silicates and oxides. Higher pressures experiments (P>5 GPa) can provide much better constraints on the pressure effects with the resolution of stress and strain measurements of currently available technology. Consequently, the reliable determination of pressure effects on plastic deformation is difficult.

Ross et al. (1979) was the first report on the pressure effect on plastic deformation of olivine. The activation volume of high-temperature creep was reported to be 13.4 cm³/mol. However, the errors in stress measurements in these experiments are very large (more than 100%), and the water content was not characterized. Therefore results in this report are unreliable. Using modified or new deformation apparatus, different values of activation volume for deformation of olivine (in the power-law creep regime) were reported. These values range from ~0 to ~28 cm³/mol (Green and Borch, 1987; Kawazoe et al., 2009; Li et al., 2006) leading to ~10 orders magnitude difference in estimated viscosity at ~400 km depth. Karato (2010) discussed the possible causes for such a wide range of results including the poor control of water content and the absence of steady-state deformation. Kawazoe et al. (2009) conducted a detailed experimental study on the pressure effects of olivine deformation using a newly developed high-pressure deformation apparatus (RDA) where all the key issues, i.e., water content, grain-size and the presence of steady-state, were examined. Operative deformation
mechanism was inferred from the study of dislocation structures. They reported the activation volume of 15–20 cm³/mol for dry olivine.

Pressure effects on diffusion were also determined for several minerals (Béjina et al., 1999; Van Orman et al., 2003; Yamazaki and Irifune, 2003). The results on lower mantle minerals are particularly relevant because there is evidence for diffusion creep in the lower mantle (Karato et al., 1995). However, the water content was not characterized in most of these studies and consequently, and hence the results are subject to large uncertainties.

4.4.2. Water weakening

The classical results by Griggs and his students demonstrated the important role of water to weaken plastic deformation of minerals (Blacic, 1972; Griggs, 1967). However, these classic studies are difficult to apply to deformation in Earth’s interior, because the appropriate parameterization was not understood and the errors in stress measurements were very large.

In 1980s, detailed studies on the influence of water on high-temperature deformation were initiated in the Paterson’s lab at the Australian National University (Chopra and Paterson, 1981; Chopra and Paterson, 1984; Karato et al., 1986; Mackwell et al., 1985). The high-resolution, low-pressure (~0.3 GPa) deformation apparatus designed by Paterson (1970) played a crucial role. Similar studies have been conducted by Mei and Kohlstedt (2000a,b) using the same type of apparatus and the same method of sample preparation and characterization. These studies confirm the earlier report by (Blacic, 1972) and provided more reliable data set on the influence of water on olivine deformation.

In order to apply laboratory results involving water-weakening effects, one needs to have an appropriate flow law parameterization. Such a parameterization was first proposed by Karato (1989a) and later extended by Karato and Jung (2003). The flow law under these conditions is given by,

\[ \dot{\varepsilon} = A \left( \frac{\sigma^n}{P_T} \right)^{m} f_{H_{2}O}(P,T) \times \exp \left( \frac{E^* + PV}{RT} \right) \]

where \( f_{H_{2}O}(P,T) \) is water fugacity that depends strongly on pressure and temperature. These papers showed that when deformation occurs at high water fugacity conditions, both water fugacity and confining pressure have large effects, and therefore the influence of these two factors need to be separated. Also Karato and Jung (2003) (see also Karato, 2006) showed that the thermodynamic behavior of water at high temperatures changes from nearly ideal gas behavior below ~0.3 GPa to highly non-ideal gas behavior above ~0.5 GPa, and consequently, if only results obtained below ~0.5 GPa were used, then these two factors cannot be separated and therefore such results cannot be extrapolated to Earth’s interior deeper than ~20 km. This indicates a major limitation of using a high-resolution but low-pressure deformation such as the Paterson apparatus for geological applications. The results by Karato and Jung (2003) to ~2 GPa show that water has a large effect of weakening olivine, and Karato (2010) suggested that the removal of water from the deep continental roots by partial melting is a likely mechanism to stabilize the continental roots.

Influence of water on high-temperature deformation of other minerals is not well characterized. For example, Post et al. (1998) extended the classic study by Griggs on quartz, but the results were not analyzed using an appropriate flow equation to separate the influence of confining pressure and that of water fugacity. Consequently, the results of this study cannot be applied to Earth’s interior with confidence. Judging from the indirect observations such as the kinetics of phase transformation and grain-growth (e.g., Kubo et al., 1998; Nishihara et al., 2006), the effects of water on deformation of transition zone minerals such as wadsleyte are likely strong but no quantitative results are available at this time. Almost nothing is known about the role of water on deformation of the lower mantle minerals such as perovskite and magnesiowüstite. These minerals dissolve large amounts of ferric iron (e.g., McCammon, 1993; McCammon, 1997), and consequently, water (proton) might not play an important role in plastic deformation as in the shallow mantle minerals. This point requires further studies.

4.4.3. Effects of partial melting

Partial melting may occur in certain regions of the Earth’s interior. In the Earth science community, there was a belief that a small amount of melt can drastically weaken the materials. This is based on some experimental studies on elastic and anelastic properties (Mizutani and Kanamori, 1964; Spetzler and Anderson, 1968). Many scientists believed that this may also apply to long-term rheology. An important breakthrough in the study of influence of partial melt on physical properties was made by Stocker and Gordon (1975) who showed that the influence of partial melting on mechanical properties depends critically on the degree of wetting of grain-boundsaries by the melt. This paper motivated a series of follow-up studies to investigate the equilibrium melt geometry in the upper mantle materials (Toramaru and Fujii, 1986; Waff, 1980; Waff and Blau, 1979, 1982) who showed that for the upper mantle materials, the wetting angle is 30–50°, and melt will be present as a connected tubules but does not completely wet the grain-boundaries. The first systematic study on the influence of partial melt on long-term plastic deformation on Earth materials was conducted by the Kohlstedt’s group. Initially, they conducted room-pressure studies (Cooper and Kohlstedt, 1984), but they also conducted studies at the confining pressure of 0.3 GPa (Hirth and Kohlstedt, 1995a,b; Mei et al., 2002). Essentially, they found only a modest effect of partial melting: addition of ~1% melt enhances deformation by ~10–30% or so. These results were interpreted assuming the equilibrium geometry of melt (Kohlstedt, 2002). According to their model, in diffusion creep, partial melting enhances deformation because mass transport in the melt is fast. In dislocation creep, deformation is enhanced by the stress concentration caused by the melt. In both cases, the influence of partial melt on deformation of olivine-rich rocks is weak at a small melt fraction because, at equilibrium, basaltic melt occurs as tubules along the triple junctions, and only a small fraction of grain-boundaries is covered with melt.

Based on these results of small direct mechanical effects of partial melting, and large effects of water to enhance deformation, and the known preference of water to go to a melt phase, Karato (1986) proposed that a small amount of partial melting will strengthen olivine-rich rocks. The concept of strengthening by partial melting was supported by the experimental study of Hirth and Kohlstedt (1995a,b), and was expanded by Hirth and Kohlstedt (1996) to develop a rheological model beneath mid-ocean ridges. It should be noted that in Hirth and Kohlstedt (1996) model, the correct parameterization of influence of water was not used, and the deformation by water-free rock was ignored (viscosity of dry olivine was assumed to be infinite). Also the competition between the mechanical weakening effects by partial melting and the hardening effects by de-watering was not discussed by Hirth and Kohlstedt (1996). Therefore, the validity of their model is questionable, particularly when such a model is applied to mid-ocean ridge dynamics (e.g., Ito et al., 1999). Karato (2008, Chapter 19) presented a more detailed analysis of the competition between weakening and hardening effects by partial melting.

Recently, Takei and Holtzman (2009a,b,c) presented a detailed analysis of deformation of a partially molten rocks and argued that deformation is more enhanced by the presence of a small amount of melt than previously considered based on laboratory studies (e.g., Kohlstedt, 2002). Their analysis involves a more detailed calculation of the stress distribution at grain-boundaries that drives diffusional flux. The cause of the difference between their results and previous models and experimental results (e.g., Kohlstedt, 2002) is not known.
The stress distribution in their model has sharp gradients that is a likely reason for the high strain-rate (a sharp stress gradient enhances diffusion e.g., Lifshitz and Shikin, 1965; Raj, 1975). However, such enhanced deformation is only transient and at steady-state, the stress distribution does not have a sharp gradient, and the creep rate agrees well with the conventional model by Raj and Ashby (1971). In any case, the enhancement of deformation by this model is only modest.

In all of these studies, the geometry of melt in a partially molten aggregate is assumed to be the same as the geometry at static equilibrium. Based on see-through experiments on analog materials, Urai (1983, 1987) reported that the geometry of fluid phase during deformation is different from the geometry at static equilibrium. The influence of stress was investigated theoretically by Hier-Majumder et al. (2004), but the influence of dynamic wetting has not been incorporated in the model of high-temperature creep in a partial melt. Jin et al. (1994) reported large effects of partial melting in the olivine–basalt system and argued that the melt wets the olivine boundaries during deformation in their experiments. The cause for the difference in the results by between Kohlstedt's and Green's groups is not known. In relation to this aspect, it should be mentioned that there is an extensive work by Holtzman and Kohlstedt on the influence of large strain on the melt distribution (Holtzman et al., 2003a,b; Holtzman et al., 2005). Such a meso-scale structural change during deformation has an important influence on melt transport.

4.5. The role of theory

Experimental studies play the most important role in the study of plastic deformation of minerals and rocks. However, theory also plays an important role. As we have seen, theory has played an important role in interpreting the experimental data and extrapolating them to deformation in Earth’s interior. We need an appropriate constitutive equation (flow law formula) to analyze the scaling issues but such a formula can be obtained only by a theoretical model. For example, Eqs. (3) and (10) are based on the theory of thermally activated motion of defects in a solid that is in chemical equilibrium with some thermo-chemical environment.

Theory is also needed when some measurements are made indirectly. An example is the measurement of stress from X-ray diffraction. By X-ray diffraction one measures the lattice spacings and hence the lattice strain. Lattice strain is directly related to the elastic strain of grains and therefore local stress at individual grains. However, in order to calculate the macroscopic stress from the local stress, one needs a theory. This issue was discussed in the previous section, 4.3.

Recently, it has become possible to calculate many properties of materials using a theory. In particular, so-called first-principle calculation has been applied to many properties including Gibbs free energy (stability of various phases) and elastic constants with great success (e.g., Karki et al., 2001; Stixrude et al., 1998; Tsuchiya et al., 2004). However, applications of this approach to plastic deformation have not been so successful. Calculating plastic properties is highly challenging for two reasons. First, in some cases particularly when dislocations are involved, the system that one needs to deal with has a low-symmetry so that one needs a large number of atoms to make any sensible calculations. This is still challenging. Second, plastic properties are controlled by many processes at various space-scales. For example, it is now possible to calculate the Peierls stress of some minerals (e.g., Carrez et al., 2007), but the relation between the Peierls stress and strain-rate is indirect. One needs to have a model for dislocation motion involving kink/jog nucleation and migration, diffusion etc. For example, if high-temperature creep rate is controlled by a simple diffusion-controlled dislocation climb, then the results of calculations of Peierls stress such as Carrez et al. (2007) have nothing to do with high-temperature creep. Therefore a detailed analysis of all possible processes of deformation is needed to discuss issues of high-temperature deformation such as the effective viscosity and the development of lattice-preferred orientation from computational approach.

More useful is a theory for mesoscopic level of physics of deformation. A good example is the calculation of plastic properties of polycrystalline aggregates from those of single crystals (e.g., Molinari et al., 1987). Such a model can also be applied to lattice-preferred orientation. An important advantage of this approach is that it is relatively easy to calculate the evolution of lattice-preferred orientation for complicated deformation history (e.g., Kaminski et al., 2004; Tommasi, 1998), but an experimental study to investigate the role of deformation history is complicated and not much has been done so far. Cordier et al. (2005) discussed various aspects of modeling at different space-scales.

5. Mechanical properties of the Earth's interior

5.1. The lithosphere

The lithosphere is a strong layer near the surface. The lithosphere includes both the crust and the shallow upper mantle. The mechanical properties of the lithosphere are important for several geological reasons. Firstly, the lithosphere is a region where various geological phenomena including mountain building and earthquakes occur. Second, the strength of the lithosphere controls the style of mantle convection. If the lithosphere is very strong, the lithosphere will not be deformed, and convection occurs only below the lithosphere. A modest strength of the (oceanic) lithosphere is a key to the operation of plate tectonics. Third, the continents have survived for ~2–3 Gyrs despite the fact that continents have moved (“drifted”) on Earth’s surface during that period. Why are the continental roots so strong? Because a large amount of literature is available on the first point (see e.g., England and Jackson, 1989; Scholz, 2002), here we discuss only the second and third points.

The key point is that plate tectonic style of mantle convection occurs only when the strength of the lithosphere is intermediate, and in fact, plate tectonics is rarely observed on terrestrial planets. More commonly observed style of mantle convection is stagnant lid convection (see Section 6.3). Given the obvious reason for the high strength of the lithosphere (low temperatures), a key question is why is the lithosphere on Earth so weak? To understand the significance of this question, let us first review a commonly accepted strength profile of the lithosphere. Soon after experimental rock deformation studies were initiated, it was recognized that rocks will yield by fracture at low confining pressures, but the mode of deformation gradually change to more distributed flow (ductile flow) at high confining pressures (e.g., Griggs et al., 1960). This is often referred to as the brittle–ductile (or brittle–plastic) transition. The strength of a rock in these regimes depends on pressure, temperature and strain-rate differently. The strength in the brittle regime is insensitive to temperature and strain-rate, whereas the strength of a rock in the ductile (or the plastic) regime is sensitive to temperature and strain-rate. Goetze was the first to integrate these points to establish a concept of strength profile of the lithosphere (Goetze and Evans, 1979). Goetze also made an assumption that the strength in the brittle regime is controlled by the propagation of faults rather than the nucleation of faults. Then the strength in this regime is controlled by the law of friction. In contrast, the strength in the ductile regime is controlled by the effective viscosity of rocks that follows the relation (3) and is sensitive to temperature (and pressure). Therefore one can construct a strength-depth profile if one knows the friction law and viscosity of rocks in the relevant materials.

Fig. 12 shows such a model for the oceanic and the continental lithosphere (Kohlstedt et al., 1995). The basic assumptions in this model are (i) the brittle strength is controlled by the friction along the fault that contains a pore fluid whose pressure is the hydrostatic...
pressure (lower than the lithostatic pressure) corresponding to its density, (ii) the strength in the ductile regime is controlled by olivine deformed by the power-law creep, (iii) the oceanic lithosphere is “dry” (no water) and (iv) the continental lithosphere is “wet” (water-saturated). Although this model captures some essence of the strength profile, this model cannot explain most of the geological processes on Earth including the operation of plate tectonics and the long-term stability of the continental lithosphere. This model predicts too high resistance for deformation of the oceanic lithosphere for plate tectonics to occur, and too weak resistance for deformation in the deep continental roots to preserve them for ∼3 Gyrs. The latter problem is due to the assumed “wet” rheology of the deep continental roots, and a possible resolution of this problem is to assume “dry” rheology with the best available experimental data on the influence of water and pressure as discussed by Karato (2010). In this model, the survival of the deep continental roots is due to the viscosity contrast between deep roots and surroundings due to the removal of water from the former by partial melting. This is an example where rheological contrast caused by compositional difference contributes to the survival of geochemical “reservoir”.

Let us discuss the issues of modest strength of the lithosphere. There have been several studies to identify the conditions under which plate tectonic style convection occurs (see Section 6.3 for more details). Essentially, in order for plate tectonic style of convection to occur, the average strength of the lithosphere must be modest, smaller than ∼200 MPa. Kohlstedt et al. (1995) model predicts ∼500 MPa which significantly exceeds this critical value. In fact, the strength of the lithosphere inferred from deformation microstructures of naturally deformed rocks suggest ∼100 MPa or less (e.g., Nicolas, 1978) with an exception of ∼300–400 MPa for the Ivrea zone (Obata and Karato, 1995).

Several models have been proposed to solve this puzzle. Goetze and Evans (1978) pointed out that under the lithospheric conditions, the Peierls mechanism rather than power-law creep may control the ductile strength. The operation of the Peierls mechanism reduces the ductile strength, but its magnitude is only modest and is not enough to reduce the strength to a level consistent with plate tectonics. Shear localization is an effective means to reduce the strength. In fact, localized deformation is commonly observed in the continental crust, and the essence of plate tectonic style of convection is the localized deformation in the surface boundary layer. Grain-size reduction by deformation and resultant operation of grain-size sensitive creep is one possible mechanism of rheological weakening (Handy, 1989; Jin et al., 1998). One advantage of this model is that this mechanism operates selectively at relatively low temperatures (<1000 K) that is consistent with the conditions in the lithosphere. However, limitations of this model are (i) even if grain-size reduction in olivine may occur, grain-growth in pure olivine is so fast that strain during weakening period will be limited, (ii) grain-size reduction occurs only after some deformation and it is not clear if such finite strain deformation can occur in the cold lithosphere. The presence of a secondly phase such as orthopyroxene will help reduce grain-growth rate, but the second problem remains. In this connection, Ohuchi and Karato (Submitted for publication) noted that the strength of orthopyroxene is much smaller than that of olivine under the lithospheric conditions, and that consequently, deformation of orthopyroxene might trigger shear localization. This idea needs to be tested against further experimental studies and microstructural observations on naturally deformed rocks. In a series of papers Bercovici and his colleagues developed a sophisticated phenomenological theory of “damage” that may promote shear localization and reduce the strength of the lithosphere (Bercovici, 1993, 2003; Bercovici and Ricard, 2005; Ricard and Bercovici, 2009).

Regenauer-Lieb et al. (2001) suggested that the presence of water will reduce the strength allowing subduction to initiate with a reasonable loading. However, the formulation of water-weakening effect in their model is not compatible with the physical model of water weakening and in addition, the cause for the presence of water in the lithosphere is not explained. In this context, Korenaga (2007) suggested that thermal cracking could transfer water down to ∼20 km depth of the oceanic lithosphere. However the presence of thermal cracking is not consistent with the known compressional state of stress in the oceanic lithosphere from focal mechanism solutions (Sykes and Sbar, 1973).

5.2. The asthenosphere

The concept of the asthenosphere, a weak layer below a strong lithosphere, was well established based on the gravity measurements (isostasy) in the mid-18th century and the discovery of the seismic low-velocity layer in early 20th century. The weakness seen for isostatic

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**Fig. 12.** A model for the lithosphere strength (Kohlstedt et al., 1995). Pure olivine mantle is assumed. Strength is for the strain-rate of $10^{-15} \text{s}^{-1}$. In the shallow part, the strength is controlled by the resistance for sliding along the faults. In the deep part, the strength is controlled by the power-law creep. “Dry” (water-free) and “wet” (water-saturated) conditions are assumed for oceanic lithosphere and continental lithosphere respectively. This model does not explain (i) the operation of plate tectonics on Earth (the oceanic lithosphere is too strong for plate tectonics to occur) nor (ii) the longevity of the deep continental lithosphere (the deep continental lithosphere is too weak to survive for more than ∼2 Gyrs).
compensation and the seismic low velocities reflects the mechanical weakness of this region. Both elastic and plastic properties of solids get weaker (softer) with temperature so a simple explanation of a weak layer (the asthenosphere) below a strong layer (the lithosphere) would simply be the high temperature in the asthenosphere. If one also accepts the presence of a stronger layer below the asthenosphere, then one needs also to have a model to explain the high strength in the deep regions of Earth.

The simplest model to explain this type of rheological layering is to invoke a temperature–depth profile that is characterized by a steep temperature gradient in the shallow region followed by a gentle temperature gradient in the deeper region (Fig. 13a). Given such a temperature profile, one can explain the presence of a weak layer between strong layers if the mechanical properties of mantle materials have appropriate range of temperature and pressure sensitivity (Fig. 13b).

The steep temperature gradient ($dT/dz = 10$ K/km or higher) in the shallow regions of Earth has been known since the days of Kelvin. The gentle temperature gradient in the deep mantle is more difficult to infer, but Shimazu (1954) inferred a gentle temperature gradient in the deep mantle from the seismological model of Earth using the Debye model of temperature dependence of elastic properties of solids (a similar work was published later by Brown and Shankland 1981). Such a temperature profile is characteristic to a convecting layer with high Rayleigh number as shown by the boundary layer theory of mantle convection (Turcotte and Oxburgh, 1967). Based on the available data on elastic properties at that time, Birch (1952) explained the presence of a low-velocity layer by a steep temperature–depth profile. In 1970s to early 1980s, a concept of anelasticity was introduced that enhances the temperature effects (e.g., Gueguen and Mercier, 1973; Minster and Anderson, 1980). Experimental studies to characterize anelasticity were obtained and applied to explain the structure of the low-velocity and high-attenuation zone (Faul and Jackson, 2005; Jackson et al., 1992, 2002). Stixrude and Lithgow-Bertelloni (2005) conducted essentially the same analysis as Birch (1952) using more complete data base, and argued that the influence of anelasticity is not large (see also Fig. 13b). Semi-quantitative models of low viscosity layer were proposed in mid-1970s (Schubert et al., 1976), but truly quantitative model of a low viscosity layer was established only recently when the first quantitative data set on the pressure- and water-dependence of creep strength of olivine was characterized (Karato and Jung, 2003; Kawazoe et al., 2009). So essentially the high-temperature gradient in the shallow mantle followed by a gentle gradient explains the presence of a low velocity and low viscosity layer at around 100 km depth without invoking partial melting.

Is there partial melting in the asthenosphere? Do we need partial melting to explain geophysical observations? A major refinement has been made in our understanding of partial melting in the last ~20 years or so, and the current understanding is that substantial (>1%) partial melting occurs only in the vicinity of mid-ocean ridges where adiabatic upwelling of hot materials occur (Plank and Langmuir, 1992). Hirschmann et al. (2009) confirmed this conclusion based on the more detailed analysis of the influence of water, and these studies showed that the amount of melt in the asthenosphere away from the mid-ocean ridge is very small (~0.1%). As I discussed already, the influence of partial melting on the mechanical properties is rather small. Given a low melt fraction, the presence of partial melt is unlikely to affect the mechanical properties of the asthenosphere significantly. Another issue about partial melting is that it is difficult to maintain a large fraction of melt in the gravity field when melt is connected. This is due to the compaction by gravity (McKenzie, 1984), and the characteristic depth scale to which one can maintain significant amount of connected melt is given by the compaction length that depends on the viscosity of the solid matrix and the permeability. The compaction length for the upper mantle materials is ~100 m (Schubert et al., 2001). Therefore it is very difficult to have a broad region of connected melt in the asthenosphere except beneath ridges where melt is continuously generated.

These were the status of understanding of the asthenosphere in the community in mid-1990s. In 1996, an interesting new observation was published (Gaherty et al., 1996). They used relatively high frequency body-waves to investigate the seismic structure of the upper mantle and found a sharp velocity change (a drop from a shallow to a deeper region) at ~60–70 km depth. A sharp velocity change causing substantial reflectivity or S-to-P conversion for a short period body wave can occur only when the velocity change occurs within the wavelength of the seismic waves (~5 km). Such a sharp velocity change cannot be explained by the classic model described above. In addition, Gaherty et al. (1996) argued that the depth of the lithosphere–asthenosphere boundary (LAB) does not depend strongly on the age of the ocean floor. These observations led us to propose a model in which the lithosphere–asthenosphere transition is caused mainly by the sharp change in the water content at ~60–70 km depth (Karato and Jung, 1998). In this model, we used a model by Karato (1986), Karato (1995) and Hirth and Kohlstedt (1996) to propose that there is a sharp change in water content at ~60–70 km depth due to the water removal by partial melting beneath the ridge (water content in the shallow part is lower than that of the deeper part).

![Fig. 13. (a) Temperature and pressure distribution in the old oceanic upper mantle. (b) Velocity-depth relationships for (i) linear anharmonic model (ah), (ii) non-linear anelasticity model (an), and (iii) a model involving water effects (after Karato, 2008). The model involving the water effect explains a sharp velocity drop at ~60–70 km, but the original model by (Karato and Jung, 1998) predicted only ~1% velocity reduction. However, the magnitude of velocity reduction can be as high as ~26% if the characteristic frequencies of relaxation are increased beyond the high frequency limit of absorption band by the addition of water.](image-url)
Water reduces viscosity and hence likely enhances anelasticity. Consequently, a sharp drop in seismic wave velocity will occur at this depth. The only problem with this model is that it predicts a rather small velocity drop. In the original version of the model, anelastic relaxation and velocity reduction are assumed to be linked directly, i.e., modifications to anelastic relaxation by water are assumed to occur within the seismic frequency band. If this is the case, then the degree of velocity drop is given by \( \Delta V = \frac{1}{2} \rho \nabla \Delta p Q^{-1} \approx (Q^{-1})^{3/2} \) and is expected to be small (\( Q \approx 100 \) and \( \alpha \approx 0.3 \)).

In this respect, recent reports of sharp and large velocity drop at LAB (e.g., Kawakatsu et al., 2009; Rychert et al., 2005; Rychert and Shearer, 2009) needs to be examined in some detail. These studies extended the previous work by Gaherty et al. (1996) and showed that not only the velocity drop is sharp but also the amplitude of velocity drop is large at LAB (~5–10%). Noting that Karato and Jung (1998) model predicts a small velocity drop, Kawakatsu et al. (2009) argued that this observation cannot be explained by a water-based model and requires the presence of partial melt in the asthenosphere. However, the model by Kawakatsu et al. (2009) has two major problems and a modified version of Karato and Jung (1998) model could better explain the LAB. Kawakatsu et al. (2009) proposed that LAB corresponds to the onset of partial melting similar to the classic model of asthenosphere (e.g., Lambert and Wyllie, 1970; Spetzler and Anderson, 1968). Knowing the problems of compaction, they argued that melt-rich layers occur as horizontal thin channels and these layers are isolated citing the results of study by Holtzman et al. (2003a,b). However, the plausibility of such a structure is questionable because, according to the study by Holtzman et al. (2003a) and Zimmerman et al. (1999), melt-rich layers are tilted from the shear plane by \( \sim 20^\circ \) (for a theoretical explanation see Katz et al., 2006). Therefore each layer in the asthenosphere should be tilted from the horizontal plane and hence the melt should be squeezed out by compaction by gravity.

There is one way out of this dilemma. In our model, it was assumed that anelastic relaxation and velocity reduction are directly linked (Karato and Jung, 1998). If we remove this assumption and allow characteristic frequencies of anelastic relaxation to be increased by water to frequencies higher than the upper limit frequency of the seismic band, then the velocity reduction can be higher than expected from \( \Delta V = \frac{1}{2} \rho \nabla \Delta p Q^{-1} \approx (Q^{-1})^{3/2} \). In fact, if the grain-boundaries are totally weakened (no strength), then the degree of velocity reduction can be calculated analytically to be \( \frac{V_{s}}{V_{r}} = \frac{1}{2} \frac{2 + 5 \nu}{7 - 4 \nu} (V: \text{relaxed velocity}, V_{r}: \text{unrelaxed velocity}, \nu: \text{Poisson's ratio}) \) (Ike, 1947). With \( \nu \approx 0.3 \), one obtains \( \sim 26\% \) reduction in seismic wave velocity. Therefore, if grain-boundary relaxation is large, it is easy to explain a large velocity reduction. Note that this model is related to partial melting, but indirectly: the role of partial melting in this model is to remove water from the rock. Consequently, in this model, the asthenosphere is soft because of a high concentration of water due to the lack of significant partial melting. This model deserves serious consideration to explain the seismological signature of LAB.

5.3. The transition zone and the lower mantle

The viscosity of the deep mantle has a number of geodynamic implications. First, if the deep mantle viscosity is very high, mantle convection would occur only in the shallow part, and the deep part will be stagnant. Such a model was proposed by McKenzie (1967). Second, stirring and mixing time-scale of geochemical reservoirs will also be affected by a large variation in viscosity (e.g., Gurnis and Davies, 1986, see also Manga, 1996). Third, if the viscous deformation in the deep mantle is difficult, then much of energy dissipation associated with mantle convection will occur in the deep mantle. If this is the case, then the secular cooling of Earth may be controlled largely by the viscous energy dissipation in the deep mantle. Although an extremely high viscosity of the lower mantle inferred by McKenzie was disputed by Goldreich and Toomre (1969) as due to the inappropriate interpretation of the non-equilibrium bulge, generally high viscosities in the deep mantle are still considered to be a valid model (see Fig. 4). However, the rheological structures of the deep mantle are not very well constrained. In the following part of this section, I will review some issues of deep mantle rheology from the point of view of mineral physics.

In the transition zone, most of minerals undergo phase transformations that affect rheological properties (e.g., Ringwood, 1991). Also in the lower mantle, although no major phase transformations occur in the most part of it, there is a large increase in pressure from \( \sim 24 \text{ GPa} \) to \( \sim 135 \text{ GPa} \), so pressure will certainly affect the rheological properties of the lower mantle. Also, the spin state of Fe changes in the lower mantle that might have some influence on the rheological properties. However, rheological properties of the transition zone and the lower mantle are difficult to study through materials science approach because pressures exceeding \( \sim 14 \text{ GPa} \) (and temperature to \( \sim 2500 \text{ K} \) or above) are needed. So the rheological properties in these deep regions of the mantle are largely unexplored, although some direct mechanical tests have been made in recent years.

Influence of phase transformations is reviewed by Karato (2008) (Chapter 15). Three aspects are particularly important. First, when a mineral changes its crystal structure, then all physical properties should change. This crystal structure effect was first discussed by Ashby and Brown (1982) for a broad range of materials. They proposed a concept of iso-mechanical group: in some cases, materials that assume the same crystal structure and similar chemical bonding follow a unified creep law relationship when appropriate normalizations are made for key variable. In such a case, the rheological properties of some mineral, for which rheological properties are unknown by direct experiments, can be inferred if a general trend is established for a class of materials to which that mineral belongs. In the analysis by Ashby and Brown (1982), all oxides are treated as a single class of material. Karato (1989c) showed, in contrast, that high-temperature plastic deformation in oxides and silicates depends strongly on crystal structure. Related issues were also discussed by Drury and Fitz Gerald (1998) (see also Chapter 15 of Karato, 2008). These results suggest that the influence of the crystal structure is generally modest but large contrasts are predicted in some cases including the higher strength of perovskite than the strength of co-existing (Mg, Fe)O. This large rheological contrast may cause strain softening and resultant shear localization as discussed by Yamazaki and Karato (2001b).

Second, when a first-order phase transformation (a phase transformation accompanied by a volume change) occurs, then internal stress-strain is created in a transforming material. This internal stress–strain may enhance deformation. This effect was first analyzed by Greenwood and Johnson (1965), and Sammis and Dein (1974) proposed that it might play an important role in the Earth’s mantle. Panasyuk and Hager (1998) presented a model that is essentially the same as the model by Greenwood and Johnson (1965), Poirier (1982) presented a related but different model than that of Greenwood and Johnson (1965) where Poirier considers that the internal strain rather than the internal stress may enhance deformation. Parmentier (1981) explored possible effects of such softening effects. Karato (2008) (Chapter 16) reviewed these models and concluded that these mechanisms may play some role only under relatively low temperature conditions, and the influence is modest, if any. So such an effect is not important in the main part of the mantle, but might play some roles in the subducting slab.

Third, and the perhaps the most important effect is the influence of the change in grain-size. When a first-order phase transformation occurs, grain-size may change. If grain-size after a transformation is much smaller than the grain-size before, this will cause rheological
weakening. The possibility of this effect was first proposed by Vaughan and Coe (1981) and Rubie (1984) extended this model to speculate its possible influence on slab deformation. In all of these models, however, the physical processes to control the grain-size after a phase transformation was not investigated quantitatively and therefore it was difficult to apply laboratory data to discuss possible effects of grain-size reduction in geological processes: even if one observes large grain-size reduction in a quick laboratory experiment, in a slower transformation in real Earth, grain-size could be larger.

Riedel and Karato (1997) presented a theoretical analysis of the physics of grain-size reduction and proposed a scaling law to calculate the plausible grain-size in the actual Earth based on the laboratory data on the nucleation and growth kinetics of relevant phase transformations. Karato et al. (2001) applied this model to the actual Earth, and discussed the nature of slab deformation in the transition zone. They found that the degree of grain-size reduction is large at low temperature, and consequently, the effective viscosity after a phase transformation may become smaller at low temperatures than at high temperatures. In this way, they explained the observed intensive deformation of slabs in the western Pacific transition zone where cold slabs subduct.

The influence of grain-size change is potentially large. If deformation occurs by grain-boundary diffusion creep (Coble creep), then viscosity is proportional to the third power of grain-size, \( \eta \propto d^{-3} \), so a change in grain-size by a factor of 10 results in the change in viscosity by a factor of \( 10^{3} \). Physical processes to control grain-size are (i) the nucleation-growth kinetics associated with a phase transformation (e.g., Riedel and Karato, 1997), (ii) the grain-growth kinetics (e.g., Solomatov et al., 2002; Yamazaki et al., 1996) and (iii) the dynamic recrystallization (e.g., Karato et al., 1980). In the most part of the lower mantle, where deformation likely occurs by diffusion creep, the first two processes are mostly important. Solomatov (1995, 2001) studied the influence of grain-size evolution on mantle convection. Kubo et al. (2009) also investigated the influence of grain-size reduction on the slab deformation in the transition zone and the lower mantle. Major limitations in these studies are the poor constraints on the rheological properties of minerals in the deep mantle and the kinetics of grain-growth. For example, the kinetics of the grain-growth reported by Yamazaki et al. (1996) and by Solomatov et al. (2002) differ significantly. These properties need to be better characterized to make progress in understanding the whole Earth dynamics and evolution.

6. Mechanical properties and mantle convection

6.1. Why wasn’t mantle convection accepted as a model for global geological activities in early 20th century?

Today, mantle convection is widely considered to play the major role in the global dynamics and evolution of Earth and other terrestrial planets (for a comprehensive review, see Schubert et al., 2001). However, the acceptance of mantle convection as a major driving mechanism of geological phenomena has not been straightforward. In view of the history of our understanding of rheological properties of Earth and thermal convection, the slow acceptance of model of mantle convection is puzzling. Here, a brief review is provided to address why the acceptance of mantle convection was so slow with particular attention to the role of our understanding of rheological properties.

Holmes (Fig. 14) was one of the first scientists to propose mantle convection as a main driving mechanism of geologic phenomena (Holmes, 1931, 1933). Citing studies by Jeffreys (1926), Holmes made a rough estimate of the minimum viscosity to prevent mantle convection (\( \sim 10^{25} \) Pa s) and used the actual viscosity of the mantle from post-glacial rebound (\( 10^{17} – 10^{22} \) Pa s) to argue that mantle convection likely occurs. A few years later, Haskell (1935a,b, 1937) calculated the (average) mantle viscosity to be \( 3 \times 10^{11} \) Pa s. In two papers by Griggs published in 1939, he emphasized that the solid mantle can deform plastically and convection likely occurs in the mantle that could explain a range of geological phenomena (Griggs, 1939a,b). Griggs cited the studies by Haskell (1935a,b, 1937), Pekeris (1935) and Vening Meinesz (1934) to support his idea. About the same time, Vening Meinesz (Fig. 15) developed the idea of mantle convection based on the measurements of gravity field in the sea including across the Pacific, East Indies (Indonesia) (Vening Meinesz, 1932, 1934) using the graviometer of his own design. He discovered a large but narrow negative gravity anomalies along the deep ocean trenches (but little anomalies across the Pacific) and interpret it as due to the presence of thickened crust by downward folding caused by mantle convection (see also Heiskanen and Vening Meinesz, 1958). The study by Vening Meinesz is particularly notable because (i) it was one of the first extensive geophysical measurements at sea preceding the later studies that led to the theory of plate tectonics, (ii) he made an extensive theoretical analysis of gravity anomalies, and (iii) particularly in Heiskanen and Vening Meinesz (1958), he developed an integrated model of convection-driven global tectonics including the geological observations summarized by Umbgrove (1938), theory of convection in the sphere by Pekeris (1935) and rock mechanics. Remarkably, in Heiskanen and Vening Meinesz (1958), they presented an extensive discussion on the role of phase transformations in the transition zone in mantle convection.

As mentioned before, by early 20th century (in 1935 or so), the theory of convective instability was well established (Jeffreys, 1926,
1930; Pekeris, 1935; Rayleigh, 1916), and an average viscosity of the mantle was well constrained (Haskell, 1935a,b, 1937). If one uses the theory by Rayleigh (1916) (modified by Jeffreys, 1926 for different boundary conditions), where convective instability is characterized by the Rayleigh number \( R_n = \frac{\Delta T \eta \kappa}{\rho C_p H^2} \); \( \alpha \): thermal expansion, \( g \): acceleration due to gravity, \( \Delta T \): excess temperature difference between the bottom and top of the fluid layer, \( H \): the thickness of the fluid layer, and one uses the simplest interpretation of Haskell's results (i.e., homogeneous viscosity), then, the Rayleigh number would be \( \sim 10^3 \) far exceeding the critical Rayleigh number \( (\sim 10^1) \). In fact, above cited authors used this type of argument to suggest mantle convection. Why, then, didn't mantle convection become a paradigm in the geological community in 1930s?

One of the important consequences of mantle convection is large horizontal motion. The most readily available geological observations are those on continents and there evidence for vertical movement is so obvious but not much evidence for horizontal motion. In this context, it is interesting to note that some Dutch scientists such as Umbgrove and Vening Meinesz who pioneered the model of mantle convection-driven geological processes were familiar with the geological and geophysical observations in the Indonesia where strong interaction between oceanic and continental regions involving horizontal motion are exposed in a clear fashion. In contrast, in other regions such as European Alps or Himalaya or America to which most of geologists/geophysicists in other countries were familiar, the geological/geophysical features are dominated by continent–continent collision and their interpretation is complicated. Indeed, the most important driving force for the development of the model of plate tectonics was the observations at sea where the evidence for horizontal motion can be clearly identified.

Another and more important reason from the point of view of physics of mantle convection is the complications in the nature of rheological properties of the mantle. I discussed that if the simplest model of mantle rheology (constant viscosity with depth) were used, one would obtain a high Rayleigh number exceeding the critical Rayleigh number for convection. However, the depth dependence of mantle viscosity was unconstrained until very recent (mid-1980s). The data of post-glacial rebound can be analyzed by a model of a thin low viscosity channel between two layers of high viscosity. If the channel thickness is \( \sim 100 \) km, then the conditions for convective instability becomes only marginal.

The role of time-dependent nature of non-elastic properties on mantle convection is another critical issue. In early 20th century it was well known that plastic flow of solids show marked time dependency (e.g., Andrade, 1916; Griggs, 1939b). Jeffreys (Fig. 16), who is one of the authorities of geophysics in the early 20th century, expressed strong opinion against mantle convection based on a model of time-dependent mechanical properties of rocks deduced from the frequency dependence of seismic wave attenuation.

3.5. Boundary layer theory

Jeffreys also contributed to the theory of thermal convection but presented a strong argument against mantle convection based on a model of time-dependent mechanical properties of rocks inferred from the frequency dependence of seismic wave attenuation. Jeffreys' approach is remarkably modern and is consistent with the currently accepted model of seismic wave attenuation \( Q \propto \omega^a \) with \( a \approx 0.3 \). If one accepts the absorption band model, and uses the rule of converting frequency dependence of attenuation to creep (Fourier transformation), one would obtain the same conclusion as Jeffreys. What is wrong? A key to solve this puzzle is the fact that for any solids, the absorption band-type behavior will no longer work at low frequencies and high temperatures because of the relaxation for “pinning” effects as discussed by Karato (1998b), Karato and Spetzler (1990) and experimentally shown by Lakki et al. (1998) (see Fig. 2). In short, Jeffreys’ arguments against mantle convection were incorrect because he extrapolated the absorption band-type model of anelasticity beyond its applicability limit.

Thus, in summary, the reasons for the slow acceptance of the model of mantle convections have various aspects. As is often discussed in the literature (e.g., Schubert et al., 2001), the lack of extensive observations at sea was an important cause for slow acceptance of mantle convection model. I should also point out that poor understanding of rheological properties also caused some skepticism to the model of mantle convection including an important discussion by Jeffreys. We need (i) a broader range of observations particularly at sea, and (ii) a better understanding of rheological properties to understand mantle convection on a firm observational and physical basis.

6.2. Boundary layer theory

One of the key steps toward the better understanding of mantle convection is the development of a theory of boundary layer convection and its applications to geological/geophysical observations. Turcotte and Oxburgh (1967) developed an analytical theory of finite amplitude convection in a fluid layer with high Rayleigh and Prandtl numbers. They showed that when both Rayleigh and Prandtl number \( Pr = \frac{\eta_1 \kappa}{\rho C_p} \) (viscosity, \( \kappa \): thermal diffusivity, \( \rho \): density) is large, turbulence is suppressed and deformation in a convecting fluid layer is localized in thin layers near the boundaries (boundary layers). Consequently, the nature of convection is characterized by the thickness of the boundary layer and the velocity with which the boundary layer moves. When the Prandtl number is large, the term containing the Prandtl number can be omitted in the momentum equation and consequently, the characteristics of convection are all determined by the Rayleigh number only. This theory predicts the
thickness of the thermal boundary layer, $h$, and the velocity of convection, $\nu$, as,
\[
b \sim \frac{1}{h} \propto Ra^{-1/3}
\]
and
\[
\nu \sim \frac{1}{h} \propto Ra^{2/3}.
\]

The temperature structure in the thermal boundary layer was also calculated by Turcotte and Oxburgh (1967), and the temperature profile corresponding to this model was tested against ocean bottom bathymetry assuming the isostatic equilibrium (Parsons and Sclater, 1977). The boundary layer theory is a milestone in the development of the model of mantle convection (for a review see e.g., Davies, 1999). It explains a number of important features of observations including (i) the thickness of the lithosphere (and its age dependence), (ii) the heat flow and its age dependence, and (iii) the velocity of mantle convection.

The surface boundary layer in this model can be identified with the lithosphere. Note, however, the lithosphere defined by the relation (11) is thermal lithosphere and the thickness of mechanical lithosphere can be different. However, because the mechanical properties are strong function of temperature, the thickness of the mechanical lithosphere follows roughly the relation (11) (for various definitions of the lithosphere see McKenzie and Bickle, 1988 and Chapter 19 of Karato, 2008). A modification to this view was proposed by Hirth and Kohlstedt (1996) and Karato and Jung (1998) based on the model by Karato (1986, 1995). These authors suggested that the water depletion beneath the mid-ocean ridge will remove water from the upwelling mantle that causes a large increase in viscosity. The water removal occurs at ~60–70 km and hence the materials above this depth beneath the mid-ocean ridge will be strong and will define the mechanical lithosphere. In this model, the thickness of the mechanical oceanic lithosphere is nearly independent of the age of the ocean floor.

In the thermal boundary layer model of mantle convection, the heat transport by convection is controlled by the heat conduction in the surface boundary layer. Therefore the heat flux carried by convection, i.e., Nusselt number ($Nu$), is related to Rayleigh number as
\[
Nu = \frac{H}{h} \propto Ra^{1/3}.
\]

This relation plays an important role in the study of thermal evolution of planets.

6.3. Influence of realistic rheological properties on mantle convection

In its simplest form of the boundary layer theory (Turcotte and Oxburgh, 1967), all features of convection (boundary layer thickness, convective velocity, heat flux) are written in terms of a single Rayleigh number corresponding to one viscosity. The rheology is assumed to be Newtonian linear rheology (i.e., strain-rate is linearly proportional to deviatoric stress). These are the over-simplification of a more complicated rheological behavior known for many solid materials. In all solids, effective viscosity (stress/strain-rate ratio) decreases with temperature, stress (above modest stress), water content, and increases with grain-size (for small grain-size). Variation of effective viscosity with these parameters can be more than several orders of magnitude. For example, if a typical value of activation energy (500 kJ/mol) is assumed then, for a given stress, the difference in strain-rate between hot (~1600 K) and a cold (~800 K) region is ~10^{16}. Extensive studies on the influence of temperature and pressure, non-linear rheology were initiated in 1980s, and Christensen (1989) provides an excellent review of these early studies (improved computational power played an important role in such studies). Among many topics in this category, I will discuss studies on the influence of temperature-dependent viscosity and on the role of phase transformations.

In relation to the influence of temperature-dependent viscosity, the first question to ask is what is the role of cold and hence strong near surface layer in mantle convection? This question was addressed by Solomatov and Moresi in the mid-1990s using both analytical and numerical approaches (Moresi and Solomatov, 1995, 1998; Solomatov, 1995; Solomatov and Moresi, 1996, 1997). Motivated largely by the observed absence of plate tectonics on other planets, they developed a refined model of convection in a fluid where viscosity is strongly dependent on temperature. In these cases, the nature of convection cannot be described by a single viscosity (Rayleigh number), but one needs to use at least two parameters such as a representative viscosity (Rayleigh number) and the temperature sensitivity of viscosity. They identified three different styles of convection as a function of these two parameters: stagnant lid regime for a large viscosity contrast, plate tectonic regime for a modest viscosity contrast, and the low viscosity contrast regime (Fig. 17) (Paul Tackley also conducted similar studies using mostly numerical simulations, see Tackley, 2000). Such a model provides an important basis to understand the diversity of tectonic styles on various terrestrial planets. This model also shows that the plate tectonic style of convection occurs only in the narrow parameter space, providing a reason for the uniqueness of plate tectonics on Earth.

In these studies where the temperature dependence of viscosity is included, it was noted that some additional assumptions are needed to reduce the strength of the lithosphere in order to realize the plate tectonic regime. Otherwise, the lithosphere is too strong and one would always have the stagnant lid convection if temperature-dependent viscosity is assumed. One obvious reason for weakening is the brittle failure. At low temperature and low pressure, a material will be deformed by brittle failure rather than viscous flow. The strength in the brittle regime is much lower than that of a viscous (ductile) regime at low temperature and pressure (see Fig. 12 in Section 5.1). However, this is not enough and one needs to introduce other mechanisms of weakening in the ductile regime at low temperature and pressure. The cause for this weakening in the ductile

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**Fig. 17.** Convection regime diagram (after Solomatov and Moresi, 1997). The nature of mantle convection is classified into three categories (I: small viscosity contrast regime, II: transitional regime, III: stagnant lid regime) using two parameters, the Rayleigh number for the hot mantle ($Ra$) and the viscosity contrast between the near surface layer and the hot mantle ($A$). Regime II corresponds to plate tectonics if some mechanisms of shear localization is introduced. This regime occurs only in the narrow parameter space.
regime is not well known as I discussed in the section of the rheology of the lithosphere. One convenient way that is often used in numerical modeling is to introduce a cut-off strength (or viscosity), called a yield strength (yield stress). Physically yield stress of any material is essentially the Peierls stress. But introducing the Peierls mechanism does not reduce the strength to this level. Other mechanisms of weakening include grain-size sensitive creep and/or the reduction of strength by a weak mineral such as orthopyroxene (see discussion in Section 5.1).

I conclude that some mechanisms of weakening are needed to realize the plate tectonic style of convection on Earth. All of these weakening mechanisms enhance shear localization. This explains the presence of transform faults in plate tectonic style of convection leading to a large fraction of the toroidal component of material motion (O’Connell et al., 1991) that is not observed in convection in a normal fluid.

Another important issue is the influence of phase transformations. The presence of a series on phase transformations in the transition zone of the mantle has been well recognized. A phase transformation changes many physical properties of minerals that may affect the style of convection. They include the latent heat, changes in density and changes in rheological properties. Most of the studies on the influence of phase transformation focused on the effects through density.

The first discussion on the influence of phase transformation is by Heiskanen and Vening Meinesz (1958) who considered the latent heat effect to change the density within a phase transition loop (see also Verhoogen, 1965). However, later studies at high Rayleigh numbers showed that the influence of distortion of phase boundary by lateral temperature gradient is more important. In all cases, the density change due to a phase transformation associated with convection is controlled by the Clapeyron slope, \( dP/dS \), entropy change associated with the phase transformation, \( \Delta V \); Volume change associated with the phase transformation. In general, a high-pressure phase has smaller molar volume and smaller entropy, and hence \( dP/dS > 0 \). However, (Navrotsky, 1980) predicted that a phase transformation to perovskite phase that occurs at \( \sim 660 \) km might have a negative Clapeyron slope because of a high entropy of perovskite caused by a weaker chemical bonding due to high Si–O coordination. This prediction was borne out by the later experimental study by Ito and Yamada (1982).

Immediately after this negative Clapeyron slope was reported, Christensen and Yuen (1984) conducted a detailed study of the influence of phase transformations on mantle convection using a simple two-dimensional geometry with a plausible range of Rayleigh number. In a realistic high Rayleigh number regime, the distortion of a phase boundary due to lateral temperature variation causes a large density variation that has an important effect on convection. If the Clapeyron slope is positive, this effect enhances convection because near a subducting slab where temperature is low, the transformation to a denser phase occurs at shallower depth giving more driving force for subduction. However, if the Clapeyron slope is negative, the effect is opposite: a phase transformation provides a resistance for slab penetration. In the mantle transition zone, the Clapeyron slope is positive in the shallow part (\(-410 \) km) but it is negative in the lower part (\(-660 \) km). Since the density change is larger for the transformation at 660 km, the latter effect is larger than the effect at 410-km discontinuity. Honda et al. (1993) and Tackley et al. (1993) extended this to three-dimensional model, and found that the influence is stronger in three dimension leading to intermittently layered convection that is consistent with the results of high-resolution seismic tomography (e.g., Fukao et al., 1992, 2001; Zhao, 2009; Zhao and Ohtani, 2009).

One complication is that, based on recent experimental studies, the absolute magnitude of the Clapeyron slope for the 660-km discontinuity appears to be much smaller than previously reported and the Clapeyron slope is also affected by some other factors such as the presence of other phases and impurities such as water (for recent developments see also Fei et al., 2004; Litasov and Ohtani, 2007; Weidner and Wang, 2000). Factors other than density change may play an important role in controlling the slab penetration through the transition zone. One such factor is the role of viscosity variation. In all of the studies cited above, the temperature dependence of viscosity was ignored. Davies (1995) examined the influence of strong slab on the penetration through the phase boundary. Karato et al. (2001) proposed that the slab strength may partly control the penetration of a slab into the lower mantle, and that the slab strength depends on temperature in an anomalous way: a cold slab may be softer than a warm slab.

6.4. Mantle convection and the thermal history of Earth and planets

Mantle convection is the most efficient way to remove the heat from Earth and other planets. Consequently, thermal evolution of a planet is controlled by the efficiency of mantle convection. In fact, one of the major reasons for Kelvin (Thomson, 1863b) to obtain unreasonably young age of Earth was the neglect of convective heat transfer in solving the thermal history of Earth (England et al., 2007; Perry, 1895a,b,c). For example, Holmes (1931, 1933) emphasized the role of mantle convection on the thermal history of Earth. However, serious thermal history modeling started when the boundary layer theory of mantle convection was well accepted and a simplified treatment of thermal history using a parameterized convection approach was developed (Davies, 1980; Schubert et al., 1979) (see also Jaupart et al., 2007 for a recent review). In this approach, the thermal history of a planet is studied by solving the global energy budget relation in which Earth is treated as a single unit characterized by one representative temperature, viz.,

\[
\frac{dT}{dt} = \frac{A^+ - A^- (\bar{T})}{CT} \tag{14}
\]

where \( \bar{T} \) is the representative temperature, \( C \) is the heat capacity, \( A^+ \) is the heat generation rate due to radioactive elements and \( A^- \) is the rate of heat loss from the surface (i.e., the Nusselt number, \( Nu \)). Heat generation due to radioactive decay, \( A^+ \), is dependent on the concentration of radioactive elements such as K, U and Th. \( A^- \) is controlled by the vigor of mantle convection and depends on the mantle temperature. The way in which \( A^- \) depends on temperature is determined by the temperature sensitivity of viscosity and the distribution of energy dissipation. In general \( A^- \) is expressed in terms of the Rayleigh number as

\[
A^- (\bar{T}) \propto Nu^2 (\bar{T}) \propto \frac{1}{Rd} (\bar{T}) \propto \frac{1}{\eta} \propto \frac{1}{\bar{T}} \tag{15}
\]

where \( \beta \) is a non-dimensional parameter. For the simplest boundary layer model, \( \beta = \frac{1}{4} \) (see Eq. (13)). For other models different values of \( \beta \) apply. Therefore, in this model, two parameters control the thermal evolution, the concentration of radioactive elements and the sensitivity of heat flux to temperature. The concentration of radioactive element is often measured as a Urey ratio, i.e., the ratio of current rate of heat generation by radioactive element to the current total heat loss rate. Given the value of \( \beta \), then for a Urey ratio, Eq. (14) can be integrated to understand the thermal history of Earth (and other planets). In doing so, the backward integration with time is a useful strategy because we know the current thermal state of Earth well. Then comparing the results of thermal regimes in the past with some (mostly geological) observations, one can obtain some hints as to the processes of thermal evolution of Earth (and other planets).

Christensen (1985) conducted a systematic study following this type of approach. The most important conclusion is that, although the geological records suggest only modestly high temperature in the Archean (e.g., Abbott et al., 1994; Arndt, 1994), the simplest model
\( (\beta = \frac{1}{3}) \) predicts much higher temperatures in the Archean. He found that the permissible value of \( \beta \) is 0.1 or less for the Urey ratio of 0.6 or less (Fig. 18). The mechanisms to reduce \( \beta \) have been debated. Christensen himself pointed out the possible importance of deformation of the lithosphere in controlling the rate of mantle convection (Christensen, 1985). Conrad and Hager (1999a,b) explored such an idea including explicitly the subduction of the lithosphere. In the model by Conrad and Hager, energy dissipation in mantle convection is assumed to occur mostly by the bending of the lithosphere at trenches.

This issue is, again, related to the issue of treating convection with highly variable viscosity. The parameterization shown in Eq. (15) implies that the nature of heat transport by convection is solely represented by a single viscosity, \( \eta(T) \). When viscosity is highly temperature sensitive, this approximation is not adequate as seen in the previous section. Conrad and Hager (1999a,b) introduced two viscosities (two Rayleigh numbers), one corresponding to mantle (mantle temperature, \( T \)), and another to lithosphere. When energy dissipation is dominated by lithosphere deformation, then the Nusselt number is largely controlled by the lithospheric Rayleigh number (lithosphere viscosity) but not by mantle viscosity. Consequently, the dependence of the Nusselt number on mantle temperature becomes weak. Korenaga (2003) refined this model by adding another element that the plate thickness may depend on mantle temperature in such a way that it increases with mantle temperature if the effective plate thickness is controlled by water depletion at mid-ocean ridges as suggested by the model by Karato (1986) and Hirth and Kohlstedt (1996). Korenaga argued that if this effect is strong, \( \beta \) will become even negative.

However, there is a common problem in both models. In these models, the strain and hence the energy dissipation by plate bending depends on the radius of curvature of plate at trenches. Both Conrad and Hager (1999a,b) and Korenaga (2003) assumed that the radius curvature is constant that leads to a singularity in the relation between lithosphere thickness and the Rayleigh number. However, a constant radius of curvature is physically unreasonable, because plate curvature for a given bending moment should depend on the flexural rigidity of plate that depends on the plate thickness (e.g., Turcotte and Schubert, 1982; see also Karato et al., 2001 and Davies, 2009). If the assumption of constant curvature of radius is removed, then there is no singularity will occur and one basis of these models will be invalidated.

Instead of (or in addition to) energy dissipation by plate bending, energy dissipation associated with slab deformation in the transition zone may also play a critical role in mantle convection. In fact, results of high-resolution seismic tomography show clear evidence of extensive slab deformation in the mantle transition zone (Fukao et al., 1992; Fukao et al., 2001). Energy dissipation associated with deep slab deformation is likely large. Also it was suggested that the effective viscosity associated with slab deformation in the transition zone has a negative or weak temperature dependence because of the temperature sensitivity of grain-size after phase transformations (Karato et al., 2001). If these results from this exploratory study are correct, then the control of plate velocity and hence cooling rate by deep slab deformation provides an alternative explanation for the weak temperature sensitivity of energy loss inferred from thermal history studies. Finally, energy dissipation in the lower mantle through grain-size sensitive creep may also lead to a similar relationship. That is, if deformation in the lower mantle occurs by diffusion creep as suggested by Karato et al. (1995), then the temperature sensitivity of grain-size will modify the effective temperature sensitivity of viscosity (Solomatov, 1996).

Rheological properties of materials under deep mantle conditions as well as physics of grain-size need to be investigated in more detail to evaluate these models.

7. Concluding remarks

This contribution provides an overview of the historical development of our understanding of mechanical properties of Earth's mantle and their implications on mantle convection. Rheological properties control of a broad range of geological problems including the origin of plate tectonics on Earth (but not on other terrestrial planets) and the nature of thermal evolution of Earth (and other terrestrial planets). However, the studies of rheological properties, and the applications of these results to geological problems are multifaceted, and the progress has not been straightforward in many cases. Important progress was often made based on the development of innovative methods and through the integration of a broad range of observations. Solution to a puzzle often lies in finding some indirect connections of various factors. For example, the puzzling observations on the asthenosphere might be due to an indirect role of partial melting to change the distribution of water. Similarly puzzling inference is the weak sensitivity of heat loss on the internal temperature of Earth. Again, a possible solution is the indirect influence of temperature through the temperature sensitivity of grain-size.

Some important issues related to the rheological properties are discussed including (i) the strength of the lithosphere (why plate tectonics on Earth?), (ii) the origin of the asthenosphere (is asthenosphere weak due to partial melting?), and (iii) the rheology of the deep mantle and its relation to the thermal evolution of Earth and other terrestrial planets (why does the heat loss rate from Earth depend so weakly on the internal temperature?). Existing models to answer these questions are evaluated and alternative models are proposed.

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