

Asymmetric shock heating and the terrestrial magma ocean origin of the Moon

By Shun-ichiro KARATO^{*1,†}

(Communicated by Ikuo KUSHIRO, M.J.A.)

Abstract: One of the difficulties of the current giant impact model for the origin of the Moon is to explain the marked similarity in the isotopic compositions and the substantial differences in the major element chemistry. Physics of shock heating is analyzed to show that the degree of heating is asymmetric between the impactor and the target, if the target (the proto-Earth) had a magma-ocean but the impactor did not. The magma ocean is heated much more than the solid impactor and the vapor-rich jets come mainly from the magma-ocean from which the Moon might have been formed. In this scenario, the similarity and differences in the composition between the Moon and Earth would be explained as a natural consequence of a collision in the later stage of planetary formation. Including the asymmetry in shock heating is the first step toward explaining the chemical composition of the Moon.

Keywords: giant impact, the Moon, magma ocean, shock heating, equation of state, Grüneisen parameter

1. Introduction

Lunar samples collected during the Apollo mission showed that the composition of the Moon is similar to the Earth's mantle, *i.e.*, the lunar composition show a smaller amount of Fe (compared to the whole Earth including the core) and relatively "depleted" nature (*e.g.*, refs. 1, 2). Hartmann and Davis³⁾ proposed that the Moon might have been formed by materials ejected from the mantle of proto-Earth by a giant impact after the core formation. If the collision is highly oblique, it will also explain the large angular momentum of the Earth-Moon system. Thus a giant impact model became a standard model for lunar formation (*e.g.*, ref. 4).

Large-scale numerical modeling approach has been used to understand the formation of the Moon (*e.g.*, refs. 5–8). However, most of these models showed difficulties in explaining the chemical composition of the Moon. For example, one of the first numerical modelings already showed that a majority of materials to become the Moon is from the impactor

if the collision occurs at a highly oblique angle⁵⁾ (see also ref. 6). This is due to the large shear of the impactor caused by an oblique collision. Recent geochemical measurements show, however, that the isotopic compositions of the refractory elements of the Moon are nearly identical to those of Earth (*e.g.*, refs. 9, 10), suggesting that a majority of the Moon is likely formed from the materials of the mantle of the proto-Earth. The inference of the major element chemistry is less robust than the estimate of the isotopic composition, but both petrological and geophysical studies suggest that the lunar mantle has higher FeO content than Earth's mantle (*e.g.*, refs. 1, 11). A viable model for the lunar origin should also explain this aspect of composition.

In order to explain the similarity in the isotopic composition, Cuk and Stewart⁷⁾ considered a nearly head-on collision of a proto-Earth with an impactor. They found that if a small impactor ($\sim 2.5\%$ of Earth mass) collides a rapidly rotating proto-Earth with high velocity (~ 30 km/s) with a relatively small grazing angle, then the ejected materials are mostly from the proto-Earth and the Moon has a composition similar to the composition of Earth's mantle. Canup⁸⁾ considered a different possibility. She showed that when a large impactor collides the proto-Earth with a modest velocity (~ 15 km/s) and

^{*1} Department of Geology and Geophysics, Yale University, U.S.A.

[†] Correspondence should be addressed: S. Karato, Department of Geology and Geophysics, Yale University, New Haven, CT 06520, U.S.A. (e-mail: shun-ichiro.karato@yale.edu).

a modest grazing angle, then the impactor and the proto-Earth will be well mixed so that Earth and the Moon will have similar composition.

In both models, the collision is not as oblique as the original model, and consequently explaining the large angular momentum of the Earth-Moon system is not straightforward.^{7),8)} In addition, there are two limitations in these models. First, these models can explain the chemical composition of the Moon only in a small parameter space. Therefore, the Moon was formed, in these models, by a highly fortuitous event. Second, these models do not explain the difference in the major element chemistry between these two planetary bodies such as the difference in the FeO (*e.g.*, refs. 1, 12).

From a more geochemical view-point, Ringwood²⁾ (on page 244) discussed that materials ejected from the giant impact to become the Moon would be mostly from the mantle of Earth because of higher temperature than the projectile although he did not discuss the role of the terrestrial magma ocean (magma ocean on the proto-Earth). Similarly, Zhang *et al.*⁹⁾ stated that the nearly identical isotopic composition of Ti may imply that materials of the Moon might come from the terrestrial magma ocean. Melting will not modify the isotopic compositions of heavy elements but will modify the major element composition (*e.g.*, refs. 13, 14). Therefore terrestrial magma ocean origin of the Moon is a plausible model because high temperature and resultant melting are the natural consequence of planetary formation. However, physical processes to justify such a model have not been examined in these previous studies.

In this paper, I will analyze one key issue related to shock heating upon a giant impact. This is the asymmetry in heating upon shock compression. As I will explain in the next section, this asymmetry is a consequence of fundamental differences in the physical mechanisms of compression between solids and (silicate) melts. Silicate melts likely ~~exists~~ exist on the surface of a growing large terrestrial planet like Earth (magma ocean), and materials in the magma ocean will be selectively heated up and ejected to the orbit surrounding the proto-Earth to become the Moon. It appears that including such physics is a first step toward the understanding of the lunar composition as a natural consequence of planetary formation. In this scenario, the fundamental differences in compressional properties of solids and silicate melts play a key to solve the puzzle of composition of the Moon. In this paper, I will present the results of calculation of heating upon a giant impact, and present a discussion

on the possible implications of the asymmetric heating on the composition of the Moon.

2. Heating by shock compression

One of the major limitations in all previous numerical models of a giant impact is the use of the same equation of state for the impactor and the proto-Earth. This is a gross over-simplification because the degree of collisional heating is highly sensitive to the materials properties particularly the compressional properties such as the bulk modulus and the Grüneisen parameter. These properties differ substantially between silicate solids and liquids.¹⁵⁾⁻¹⁷⁾ In the later stage of planetary formation when the Moon was formed, the proto-Earth likely had a magma-ocean on its surface but the impactor was likely a completely solid planet. In these cases, proto-Earth materials and impactor materials will be heated differently, leading to a substantially different consequence for material ejection and hence the composition of the Moon that might have been formed from the ejected materials.

To illustrate the importance of the differences in compressional properties in the collisional heating, I will consider a simple case where two bodies (one liquid and another solid) collide at a planar interface. The collision of two bodies generates pressure and volumetric strain in each body (Fig. 1), leading to heating. Thermodynamics of heating upon collision can be formulated exactly as far as a sufficiently intense collision occurs so that the Hugoniot elastic limit (HEL) is exceeded. For a typical silicate mineral this is a case when the collision velocity exceeds ~ 0.5 km/s. The collision velocity associated with a giant impact is likely close to the escape velocity of the proto-Earth (~ 10 km/s for Earth¹⁸⁾), so this condition should be well satisfied.

Under these conditions, using the conservation of mass, momentum and energy one can obtain the Rankine-Hugoniot equation of state (*e.g.*, ref. 19),

$$E - E_o - \frac{1}{2}(P + P_o)(V_o - V) = 0 \quad [1]$$

where E is internal energy, P is pressure and V is volume, and the suffix “o” means the initial state. Differentiating Eq. [1], and combining with the thermodynamic identities, $dE = TdS - PdV$ and $TdS = C_v dT - \frac{\alpha T}{\rho} dP$, one obtains,²⁰⁾

$$dT = \left\{ -\frac{T\gamma}{V} + \frac{1}{2C_v} \left[(P - P_o) + (V_o - V) \frac{dP}{dV} \right] \right\} dV \quad [2]$$

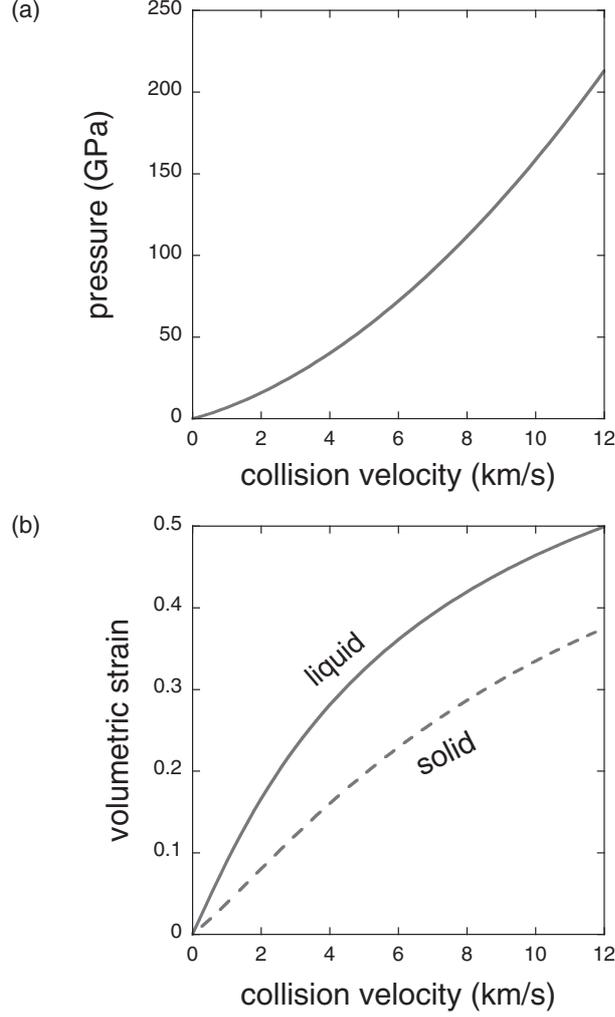


Fig. 1. (a) Collision velocity versus pressure. (b) Collision velocity versus volumetric strain. Collision of two bodies at a planar interface is considered. The pressure was calculated using the continuity of displacement and pressure at the interface and the conservation laws (mass, momentum and energy).²⁹⁾ The collision is assumed to occur above the Hugoniot-elastic limit (HEL). The volumetric strain is calculated using the Rankine-Hugoniot equation of state. The parameters used are shown in Table 1.

where γ is the Grüneisen parameter, C_v is the specific heat, V is the volume at pressure P , and V_o is the volume at pressure P_o . Pressure P is related to the molar volume, V , through the equation of state.

This relation means that if one knows the Grüneisen parameter (as a function of density) and the equation of state (*i.e.*, $P = P(V)$), then one can calculate the temperature increase upon compression. The first term of Eq. [2] represents the adiabatic heating and the second term corresponds to the

heating due to the excess entropy produced by shock deformation. Both of these terms are sensitive to material properties such as the bulk modulus and the Grüneisen parameter (including its volume dependence) and hence the temperature increase due to a collision depends strongly on the compressional properties of materials.

The Grüneisen parameter may be written as $\gamma = \left(\frac{\partial \log T}{\partial \log \rho}\right)_{ad}$ that indicates that the Grüneisen parameter expresses the degree of heating by compression under the adiabatic condition. The recent theoretical and experimental studies showed that the behavior of the Grüneisen parameter upon compression is largely different between liquids and solids (*e.g.*, refs. 15–17): the Grüneisen parameters for silicate liquids are much larger than those for solids at high compression. This implies higher degree of heating for liquids than for solids. Jing and Karato¹⁵⁾ explained this difference by a fundamental difference in the mechanism of compression between these two classes of materials.

The Rankine-Hugoniot equation of state can be cast into the following relation if one uses an empirical linear relationship between the particle velocity and the velocity of the motion of the shock front ($U_s = C_o + \lambda u_p$ where U_s is the velocity of the shock front and u_p is the particle velocity), viz. (*e.g.*, ref. 19),

$$P - P_o = C_o^2 \frac{V_o - V}{[V_o - \lambda(V_o - V)]^2} \quad [3]$$

where $C_o = \sqrt{\frac{K_o}{\rho_o}}$ is the bulk sound velocity, and $\lambda = \frac{(dK/dP)_o + 1}{4}$. Again, there is a large difference in bulk moduli (*i.e.*, the bulk sound velocity) between solids and non-metallic liquids. Bulk moduli of solids (at zero pressure) vary from one material to another, whereas the bulk moduli of non-metallic liquids are similar and much smaller than those of corresponding solids.¹⁵⁾ Jing and Karato¹⁵⁾ attributed this to the fundamental difference in the compression mechanisms of these materials.

Using the relation [3], Eq. [2] can be integrated to yield,

$$T(x) = \exp\left(-\int_1^x \frac{\gamma(\theta)}{\theta} d\theta\right) \times \left[T_o - \frac{\lambda C_o^2}{C_v} \int_1^x d\xi \left\{g(\xi) \cdot \exp\left[\int_1^\xi \frac{\gamma(y)}{y} dy\right]\right\}\right] \quad [4]$$

with $x = \frac{V}{V_o}$ and $g(x) = \frac{(1-x)^2}{[1-\lambda(1-x)]^3}$. I have calculated the temperature increase upon a collision of liquid and solid at a planar interface for various collision

Table 1. Material properties used in the computation of shock heating

	ultramafic melt	olivine
k_o (GPa)	60 (GPa)*	160 (GPa)*
λ ($= \frac{(dk/dp)_o+1}{4}$)	1.3	1.3
γ_o	1.5	1.5
q^{**}	-2- -1	0-1

k_o : Bulk modulus at the reference state.

γ_o : Grüneisen parameter at the reference state.

*Zero pressure values of bulk modulus of ultramafic melt and olivine are ~ 20 GPa and ~ 120 GPa respectively, but collisional effects penetrate into the deep interior. I assumed the average pressure of ~ 10 GPa for materials that are subject to shock compression.

** $\gamma = (\frac{V}{V_o})^q$, q is negative for non-metallic liquids (*e.g.*, refs. 15, 16, 30), but positive for solids (~ 1 , *e.g.*, ref. 31).

velocities (Fig. 2). For a liquid, I assumed the initial temperature of 2,000 K, whereas for solids 1,500 K was assumed (slightly different values of these temperatures would not change the main conclusions). Some of the key parameters used in this calculation are summarized in Table 1. The temperature increase in liquid is much higher than that in solid. This is partly due to the higher volumetric strain of liquid than solid (due to the smaller bulk modulus of liquids than solids), but the difference in the volume dependence of the Grüneisen parameter also plays an important role. One might consider that above the HEL, any materials behave like fluids and there is no distinction between liquids and solids. This is not correct. Even above the HEL, the long-range order is present in a solid material (because deformation occurs by simple shear by the propagation of dislocations and/or stacking faults above the HEL) and the compression mechanism of such a material is different from that of a liquid.

When temperature reaches certain values, phase transformations will occur (*e.g.*, from solid to liquid, from liquid to gas). The melting and the vaporization temperature depend strongly on pressure. Detailed phase diagrams of relevant materials for a broad range of pressures and temperatures (P to a few hundred of GPa, and T to $\sim 30,000$ K) are not available, but Melosh²¹) calculated a phase diagram for SiO₂ for these conditions and compared the results with experimental observations by ref. 22. If the phase diagram of SiO₂ can be used as a guide, we expect that at the peak pressure and temperature corresponding to the collision velocity of ~ 10 km/s (the escape velocity of the proto-Earth) or higher, materials in the magma ocean will be in the super-

critical fluid which, upon adiabatic cooling, will become a vapor phase. In contrast, solid planet will remain solid (Fig. 2). A vapor phase in general has a larger molar volume than the condensed phases and hence upon shock heating and subsequently cooling, materials that was initially in the magma ocean will expand their volume substantially, forming vapor jets.²³⁾

3. Speculations on the fate of shocked materials

The height to which the ejected materials are transported initially and the initial velocity (relative to the planet to which collision occurs) of shocked materials play an important role in controlling the fate of ejected materials⁴⁾ (Fig. 3). If the height and initial velocity are too large, these materials escape from the proto-Earth. If the height and/or the initial velocity are too low they come back to the proto-Earth (re-impact). Only when the height and velocity have the appropriate values, the ejected materials will orbit the proto-Earth from which the Moon could be formed.

In previous models of giant impact with oblique collision, materials of the impactor are highly sheared and the sheared materials go into the orbit surrounding the proto-Earth.^{5,6)} A giant impact will also produce vapor jets (made of highly heated materials in the magma ocean on the proto-Earth) that will go into the orbit surrounding the proto-Earth.²³⁾ Consequently, the composition of the Moon will be controlled by the competition between the shear of the impactor and the effects of vapor jets. The relative contribution of shearing and vapor jets may be estimated by comparing two forces (Fig. 4): the force due to the pressure gradient associated with a jet and the force (per unit volume) associated with the shear. The ratio is given by $f_{jet}/f_{shear} = \frac{\Delta p}{\Delta x} \frac{\Delta t}{\Delta(\rho v)} \frac{1}{\xi}$ where ρ is the density of the impactor ($\sim 3,000$ kg/m³), v is the collision velocity (~ 10 km/s), Δt is collision time (~ 1000 sec⁶⁾), ΔP is the pressure change due to vaporization (200 GPa), Δx is the space scale at which pressure changes ($\sim 1,000$ km), and ξ is a parameter that depends on the collision geometry ($0 < \xi < 1$) (*e.g.*, ref. 23). Inserting plausible values listed above, one finds $f_{jet}/f_{shear} \approx \frac{10}{\xi} \gg 1$, suggesting that vapor jet will dominate. The actual difference in the motion of materials is also related to the resistance of materials for deformation. Since the resistance for motion (deformation) is much less for the gas than for the solid (impactor), I conclude that materials in the vapor jet are likely ejected away from the proto-

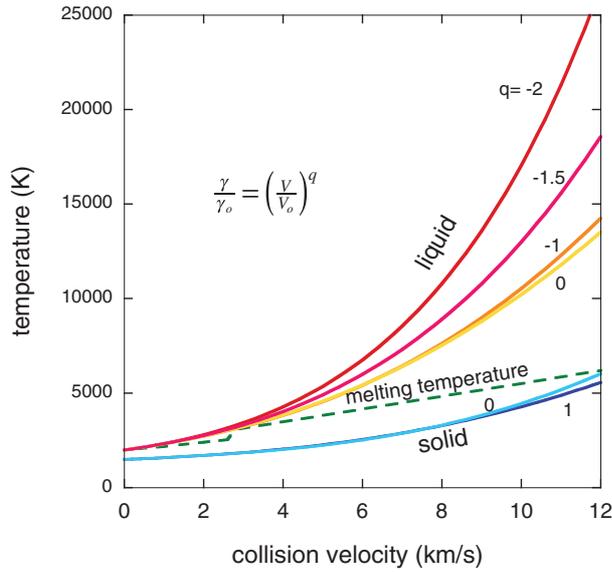


Fig. 2. Collision velocity versus temperature. The temperature increase by a collision is calculated using the relation [1] in the text. This temperature corresponds to temperature at the highest shock pressure. Parameters used are summarized in Table 1. γ : the Grüneisen parameter, V : volume, q : a non-dimensional parameter in relation $\frac{\gamma}{\gamma_0} = \left(\frac{V}{V_0}\right)^q$ ($q \sim 1$ for solids, $q < 0$ for liquids).

Earth more than the materials from the deformed impactor and hence contribute more to the Moon.

The composition of a magma ocean can be estimated from the phase diagram (element partitioning) of peridotite and the principles of the isotopic fractionation. I use the experimental data on Fe/Mg partitioning between olivine and ultramafic melt¹⁴⁾ to estimate the Mg# of magma ocean for a various degree of melting. Inferred value of Mg# of the Moon ($\sim 83 \pm 5$) can be explained by a broad range of degree of partial melting (~ 20 – 80%) or fractional crystallization. The degree of chemical segregation upon melting depends on the process of melting. Melting by a giant impact and partial melting will have somewhat different consequence on chemical segregation. However, the experimentally observed Fe/Mg partitioning between the ultramafic melts and olivine implies that melt will always be more FeO rich than co-existing solids under these conditions. In contrast to the major element composition, isotopic fractionation elements is much less affected by partial melting (*e.g.*, ref. 13). However, the chemical composition of the Moon can be somewhat different from the composition of the materials ejected by the impact because the processes of condensation from the gas-liquid mixture can

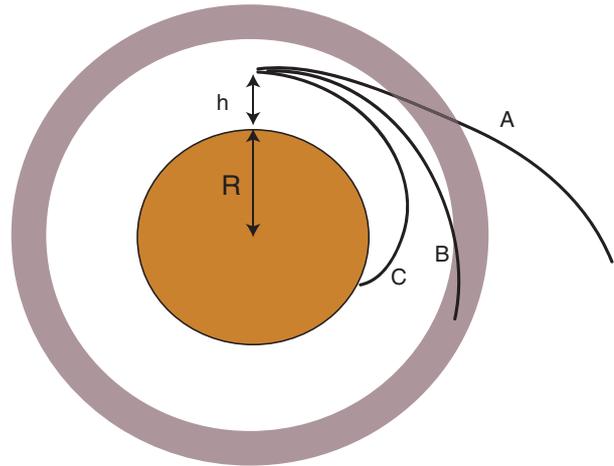


Fig. 3. A schematic diagram showing possible paths of materials ejected at a certain height. Only a fraction of materials goes to the orbit (shaded region) from which the Moon was formed. The fate of ejected materials depends on the ratio h/R and materials with only for modest value of h/R and velocity will become the source of the Moon.

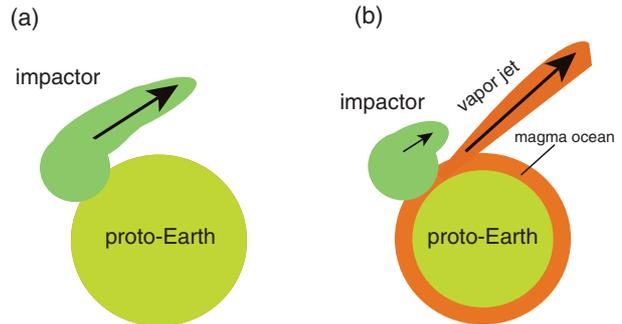


Fig. 4. Schematic drawing of processes of ejection of materials upon a giant impact. (a) A case where the proto-Earth does not have a magma ocean. (b) A case where the proto-Earth has a magma ocean.

modify the composition (*e.g.*, ref. 24). The present model could also explain the presence of a small Fe-rich core,²⁵⁾ if the influence of reduction at high temperatures is included.

3. Summary and discussions

The current model of planetary formation suggests that a large degree of heating occurs in the late stage of planetary formation. The efficient heating in this stage is a result of quick and hence efficient conversion of large gravitational energy to heat due to the collisions of relatively large objects. Consequently, high degree of heating will occur leading to melting (magma ocean formation) in a large planet (larger than Mars, *e.g.*, ref. 26), but not

on a smaller one. Therefore the proto-Earth is likely covered with a magma ocean, but an impactor (Mars size?) is likely a completely solid planet. If such a proto-Earth is hit by a smaller solid planet, heating occurs mostly in the magma ocean. Vaporized materials from the magma ocean will go into the orbit and finally form the Moon. In this scenario, composition of the Moon including the similarity in the isotopic composition and the dissimilarity in the major element composition is a natural consequence of a giant impact that occurs in the later stage of planetary formation.

Because a majority of vaporized materials from which the Moon could have formed is from the magma ocean, highly oblique collision can also form the Moon from the proto-Earth. Consequently, explaining the large angular momentum of the Moon-Earth system in this model is more straightforward than the models by Cuk and Stewart⁷⁾ or by Canup.⁸⁾

One issue in this model is the cooling time-scale of the magma ocean. Cooling time scale of a magma ocean depends on its depth extent (and hence temperature) as well as the surface conditions (presence or not of the thick atmosphere) (*e.g.*, ref. 27). Some of the hot magma oceans in the deep mantle might have a short life-time (~ 1000 years or less). However, Abe²⁷⁾ showed that a shallow and relatively cool magma ocean (~ 300 km depth) would survive for ~ 100 – 200 Myrs due largely to the presence of the thick atmosphere. Since the timing of the Moon forming giant impact is \sim several 10s Myrs after the formation of the Solar System (*e.g.*, ref. 28), it is likely that a shallow magma ocean existed when a Moon forming giant impact occurred. A 300 km thick magma ocean on Earth will have $\sim 4 \times 10^{23}$ kg of mass in comparison to the mass of the Moon of 7×10^{22} kg, and hence has enough mass to be a source of the Moon if a substantial fraction of the materials of the magma ocean became the Moon.

Another limitation of the present work is that the influence of the entropy change upon the vaporization on the temperature increase is not included. This effect (effect of latent heat of vaporization) will reduce the temperature by shock compression and hence the actual temperature increase upon shock compression will be lower than those shown by my present calculation. Nevertheless, the conclusion that the pre-existing magma ocean will be substantially more heated than the completely solid impactor will remain unchanged.

Although the analysis I presented in this paper provides a possible new way to understand the origin

of the Moon, the details of the model particularly the influence of realistic geometry of collisions and the mechanical aspects such as the angular momentum transfer are not studied in this paper. Numerical modeling incorporating the different compressional properties of materials will be needed to assess the plausibility of the present model. Key materials properties in such modeling include the difference in the compressional properties between molten and solid materials, and the influence of latent heat of vaporization.

Acknowledgements

This study was partly supported by National Science Foundation. I thank Lindy Elkins-Tanton, Kazuhito Ozawa, Hiroko Nagahara, Kaveh Pahlevan, and Zhengrong Wang for discussions. Comments by two anonymous reviewers were helpful in improving the paper.

References

- 1) Wieczorek, M.A., Jolliff, B.L., Khan, A., Pritchard, M.E., Weiss, B.P., Williams, J.G., Hood, L.L., Richter, K., Neal, C.R., Shearer, C.K., McCallum, I.S., Tompkins, S., Hawke, B.R., Peterson, C., Gillis, J.J. and Bussey, B. (2006) The composition and structure of the lunar interior. *Rev. Mineral. Geochem.* **60**, 221–364.
- 2) Ringwood, A.E. (1979) *Origin of the Earth and Moon*. Springer Verlag, Berlin.
- 3) Hartmann, W.K. and Davis, D.R. (1975) Satellite-sized planetesimals and lunar origin. *Icarus* **24**, 504–515.
- 4) Stevenson, D.J. (1987) Origin of the moon—The collision hypothesis. *Annu. Rev. Earth Planet. Sci.* **15**, 271–315.
- 5) Benz, W., Slattery, W.L. and Cameron, A.C.W. (1986) The origin of the moon and the single-impact hypothesis I. *Icarus* **66**, 515–535.
- 6) Canup, R.M. (2004) Simulation of a late lunar-forming impact. *Icarus* **168**, 433–456.
- 7) Cuk, M. and Stewart, S.T. (2012) Making the Moon from a fast-spinning Earth: A giant impact followed by resonant despinning. *Science* **338**, 1047–1052.
- 8) Canup, R.M. (2012) Forming a Moon with an Earth-like composition via a giant impact. *Science* **388**, 1052–1055.
- 9) Zhang, J., Dauphas, N., Davis, A.M., Leya, I. and Fedkin, A. (2012) The proto-Earth as a significant source of lunar material. *Nat. Geosci.* **5**, 251–255.
- 10) Wiechert, U., Halliday, A.N., Lee, D.-C., Snyder, G.A., Taylor, L.A. and Rumble, D. (2001) Oxygen isotopes and the Moon-forming giant impact. *Science* **294**, 345–348.
- 11) Khan, A., Connolly, J.A.D., MacLennan, J. and Mosegaard, K. (2007) Joint inversion of seismic and gravity data for lunar composition and

- thermal state. *Geophys. J. Int.* **168**, 243–258.
- 12) Taylor, R.S., Pieters, C.M. and MacPherson, G.J. (2006) Earth-Moon system, planetary science, and lessons learned. *In* *New Views of the Moon* (eds. Jolliff, B.L., Wieczorek, M.A. and Shearer, C.K.). Mineralogical Society of America, Chantilly, pp. 657–704.
 - 13) Allègre, C.J. (2008) *Isotope Geology*. Cambridge University Press, Cambridge.
 - 14) Mibe, K., Fujii, T., Yasuda, A. and Ono, S. (2006) Mg-Fe partitioning between olivine and ultramafic melts at high pressures. *Geochim. Cosmochim. Acta* **70**, 757–766.
 - 15) Jing, Z. and Karato, S. (2011) A new approach to the equation of state of silicate melts: An application of the theory of hard sphere mixtures. *Geochim. Cosmochim. Acta* **75**, 6780–6802.
 - 16) Mosenfelder, J.L., Asimow, P.D. and Ahrens, T.J. (2007) Thermodynamic properties of Mg_2SiO_4 from shock measurements to 200 GPa on forsterite and wadsleyite. *J. Geophys. Res.* **112**, doi:10.1029/2006JB004364.
 - 17) Asimow, P.D. (2012) Shock compression of pre-heated silicate liquids: Apparent universality of increasing Grüneisen parameter upon compression. *In* *Shock Compression of Condensed Matter* (eds. Elert, M.L., Buttler, W.T., Borg, J.P., Jordan, J.L. and Vogler, T.J.). American Institute of Physics, Melville, New York, pp. 887–890.
 - 18) Agnor, C.B., Canup, R.M. and Levison, H.F. (1999) On the character and consequences of large impacts in the late stage of terrestrial planet formation. *Icarus* **142**, 219–237.
 - 19) Duvall, G.E. and Fowles, G.R. (1963) Shock waves, *In* *High Pressure Physics and Chemistry* (ed. Bradley, R.S.). Academic Press, New York, pp. 209–291.
 - 20) Carter, W.J., Fritz, J.N., Marsh, S.P. and McQueen, R.G. (1975) Hugoniot equation of state of the lanthanides. *J. Phys. Chem. Solids* **36**, 741–752.
 - 21) Melosh, H.J. (2007) A hypercode equation of state for SiO_2 . *Meteorit. Planet. Sci.* **42**, 2079–2098.
 - 22) Lyzenga, G.A., Ahrens, T.J. and Mitchell, A.C. (1983) Shock temperatures of SiO_2 and their geophysical implications. *J. Geophys. Res.* **88**, 2431–2444.
 - 23) Melosh, H.J. and Sonett, C.P. (1986) When worlds collide: Jetted vapor plumes and the Moon's origin. *In* *Origin of the Moon* (eds. Hartmann, W.K., Phillips, R.J. and Taylor, G.J.). The Lunar and Planetary Institute, Houston, pp. 621–642.
 - 24) Pahlevan, K., Stevenson, D.J. and Eiler, J.M. (2011) Chemical fractionation in the silicate vapor atmosphere of the Earth. *Earth Planet. Sci. Lett.* **301**, 433–443.
 - 25) Weber, R.C., Lin, P.-Y., Garner, E.J., Williams, Q. and Legnonné, P. (2011) Seismic detection of the lunar core. *Science* **331**, 309–312.
 - 26) Sasaki, S. and Nakazawa, K. (1986) Metal-silicate fractionation in the growing Earth: energy source for the terrestrial magma ocean. *J. Geophys. Res.* **91**, 9231–9238.
 - 27) Abe, Y. (1997) Thermal and chemical evolution of terrestrial magma ocean. *Phys. Earth Planet. Inter.* **100**, 27–39.
 - 28) Kleine, T., Touboul, M., Bourdon, B., Nimmo, F., Mezger, K., Palme, H., Jacobsen, S.B., Yin, Q.-Z. and Halliday, A. (2009) Hf-W chronology of the accretion and early evolution of asteroids and terrestrial planets. *Geochim. Cosmochim. Acta* **73**, 5150–5188.
 - 29) Rinehart, J.S. (1975) *Stress Transients in Solids*. HyperDynamics, Santa Fe.
 - 30) Stixrude, L. and Karki, B.B. (2005) Structure and freezing of MgSiO_3 liquid in Earth's lower mantle. *Science* **310**, 297–299.
 - 31) Anderson, O.L. (1996) *Equation of State of Solids for Geophysics and Ceramic Sciences*. Oxford University Press, Oxford.

(Received Dec. 23, 2013; accepted Jan. 20, 2014)