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Effective thermal expansivity of Maxwellian oceanic lithosphere

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Abstract

The thermal expansivity of oceanic lithosphere is a key mineral physics parameter that controls the rate of seafloor subsidence. Because of strongly temperature-dependent mantle rheology, effective expansivity for lithosphere as a whole could be substantially lower than indicated by mineral physics data. Viscoelastic modeling indicates that this reduction in expansivity could be as high as ∼15–30% for a plausible range of mantle viscosity. Though brittle relaxation by thermal cracking is likely to alleviate this reduction, tension cracking results in fractured lithosphere with finite crack porosity, thereby prohibiting the recovery of fully equilibrated density. Even with complete brittle relaxation, effective thermal expansivity could still be lower by up to ∼20%.

Keywords: thermal stress; viscoelasticity; seafloor subsidence

1. Introduction

Calculating the thermal structure of oceanic lithosphere with variable material properties has become a common exercise in recent years [1–4]. While pressure dependency is not important for lithospheric depth scales, temperature dependency can be significant because temperature varies over more than 1000 K within the lithosphere. When material properties are only temperature dependent, the equation of heat conduction is

$$\rho(T)C_P(T)\frac{\partial T}{\partial t} = \nabla\cdot(k(T)\nabla T), \tag{1}$$

where T is temperature, t is time, and ρ, C_P, and k denote density, specific heat, and thermal conductivity, respectively. The temperature dependency of density is controlled by volumetric thermal expansivity α as

$$\rho(T) = \rho_0\exp\left(-\int_{T_0}^{T}\alpha(T)\mathrm{d}T\right), \tag{2}$$

where ρ_0 is reference density at $T=T_0$. When temperature increases from 300 K to 1600 K in the lithosphere, $\alpha(T)$ increases by ∼30% [5], $C_P(T)$ increases by ∼60% [6], and k decreases by ∼50% [2]. In terms of solving the heat conduction Eq. (1), a detailed understanding of $\alpha(T)$ is not important because the corresponding variation of $\rho(T)$ is only a few percent. However, $\rho(T)$ (and thus $\alpha(T)$) plays a critical role in evaluating the success of a particular thermal model by comparing model prediction with seafloor subsidence. The subsidence of seafloor after its creation at mid-ocean ridges is caused by density increase associated with thermal contraction, and a 30% difference in α, for example, roughly corresponds to as much as 1 km difference for the predicted depth of 100 Ma seafloor.

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Seafloor depth and heat flow are two primary observational constraints on the thermal structure of oceanic lithosphere [7,8], and because heat flow data are rather noisy, seafloor subsidence has been the most important constraint. Thus, incorporating the precise knowledge of $\alpha(T)$ in model calculation is essential when refining our understanding of oceanic lithosphere by combining mineral physics data and geophysical observations.

The direct use of $\alpha(T)$ from mineral physics experiments, however, may not be warranted for oceanic lithosphere, the evolution of which is characterized by rapid cooling with a large temperature contrast. Because of strongly temperature-dependent mantle rheology, oceanic lithosphere may not be able to attain complete thermal contraction. Whereas vertical contraction is always guaranteed because the seafloor is a free-moving surface, horizontal contraction can be suppressed if viscous relaxation is inefficient. In other words, the "effective" thermal expansivity of oceanic lithosphere could be lower than indicated by mineral physics data. The purpose of this paper is to lay out a basic theoretical formulation for the effective thermal expansivity of a viscoelastic medium subject to instantaneous cooling. For a range of plausible mantle rheology, I will show that this viscoelastic effect is not trivial. The possibility of 'brittle' relaxation and its consequence are also discussed.

2. Theoretical formulation

The thermal contraction of oceanic lithosphere is a complicated subject because mantle rheology is characterized by both brittle and ductile deformations in the uppermost mantle [9]. I thus begin with a very simple viscoelastic model, which will then be elaborated to approach a more realistic situation. For the purpose of theoretical development here, the temperature dependency of α, C_P, and k is not important, so these material properties are assumed to be constant for the sake of simplicity. Then, thermal diffusivity, $\kappa(=k/(\rho C_P))$, is also constant.

I first consider a one-dimensional viscoelastic model with the following initial and boundary conditions. For $t<0$, a viscoelastic medium with depth D and width L (in both horizontal directions) has a uniform temperature of T_i, and its surface temperature is suddenly fixed to $T_s(<T_i)$ at $t=0$. All other boundaries are insulating. For $t<\sim 0.1D^2/\kappa$, the subsequent evolution of the temperature field is well approximated by the well-known half-space cooling solution [10]:

$$T(z,t) = T_s + (T_i - T_s)\,\mathrm{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right). \tag{3}$$

The top boundary is free-moving, and all other boundaries are fixed to have zero displacement. This boundary condition is most appropriate when quantifying the magnitude of thermal stress in oceanic lithosphere [11]. I will return to this issue later.

To study stress perturbations caused by cooling, we can safely neglect the effect of gravity (which is important for the reference pre-stressed state), so the appropriate momentum balance is the following:

$$\sigma_{ij,j} = 0, \tag{4}$$

where σ_{ij} is the stress tensor. For a Maxwell viscoelastic body, the constitutive relation including thermal stress is given by [12]:

$$\dot{\sigma}_{ij} + \frac{\mu}{\eta}\sigma_{ij} = 2\mu\dot{\varepsilon}_{ij} + \delta_{ij}\left[\lambda\dot{\varepsilon}_{kk} - \alpha K\delta\dot{T} + K\frac{\mu}{\eta}(\varepsilon_{kk} - \alpha\delta T)\right], \tag{5}$$

where ε_{ij} is the strain tensor, δT is deviation from the initial temperature $(=T-T_i)$, λ and μ are Lamé's elastic constants, η is viscosity, and K is bulk modulus $(=\lambda+2\mu/3)$. The dot denotes differentiation with respect to time. Note that α in the above is volume (not linear) thermal expansivity.

In the instantaneous relaxation limit $(\eta\rightarrow 0)$, the constitutive relation reduces to

$$\sigma_{ij} = \delta_{ij}K(\varepsilon_{kk} - \alpha\delta T). \tag{6}$$

For the aforementioned boundary condition, this gives rise to zero thermal stress, and the vertical displacement at the surface is given by

$$u_z(z=0) = \alpha\int_0^D \delta T\,\mathrm{d}z. \tag{7}$$

That is, full thermal contraction is achieved even though only the top surface is free to move. On the other hand, in the zero relaxation (or purely elastic) limit $(\eta\rightarrow\infty)$, the constitutive relation becomes

$$\sigma_{ij} = \delta_{ij}\lambda\varepsilon_{kk} + 2\mu\varepsilon_{ij} - \delta_{ij}\alpha K\delta T, \tag{8}$$

and the corresponding vertical displacement is

$$u_z(z=0) = \frac{K}{\lambda+2\mu}\alpha\int_0^D \delta T\,\mathrm{d}z. \tag{9}$$

Thermal contraction is reduced by the factor $K/(\lambda+2\mu)$, which is ~ 0.55 for the uppermost mantle. It is interesting to note that this factor is greater than 1/3.

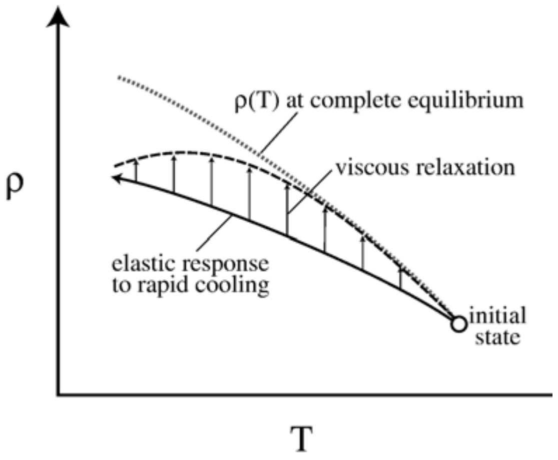

Fig. 1. Role of viscous relaxation in thermal contraction. $\rho(T)$ provided by mineral physics experiments assumes complete equilibrium (dotted), which may not be attained in the cooling of oceanic lithosphere. Minimum density increase is given by the purely elastic limit (solid, Eq. (10)). Temperature-dependent viscosity controls the rate of viscous relaxation to equilibrium, so the colder part of lithosphere may not be able to fully contract (dashed).

We may express these two limits in terms of effective thermal expansivity as

$$\frac{K}{\lambda+2\mu} \leq \frac{\alpha_{\mathrm{eff}}^{\mathrm{M}}}{\alpha} \leq 1, \tag{10}$$

where the superscript M indicates that this effective expansivity corresponds to a purely viscoelastic medium.

The $\alpha_{\mathrm{eff}}^{\mathrm{M}}$ of oceanic lithosphere must lie somewhere between these bounds (Fig. 1), and it is uniquely defined once the elastic and viscous parameters are specified. Muki and Sternberg [13] derived an analytical solution for this viscoelastic problem in the case of temperature-dependent Newtonian viscosity. Korenaga [11] developed a numerical code to handle a more general form of viscosity as well as arbitrary boundary conditions. Though the ductile deformation of mantle materials is characterized by both diffusion (Newtonian) and dislocation (non-Newtonian) creep, it is found that diffusion creep is the most dominant mechanism for the thermo-viscoelastic

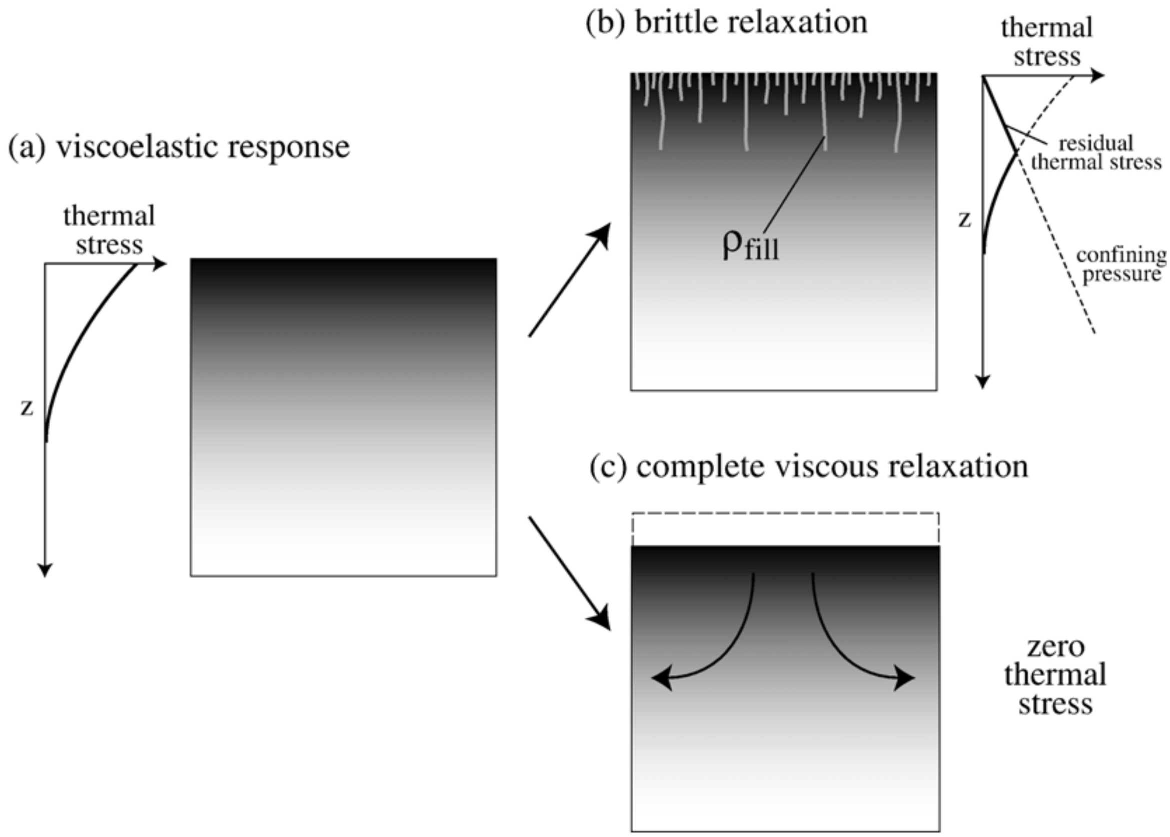

Fig. 2. Concept of 'brittle' relaxation by thermal cracking. (a) Without brittle failure, viscoelastic response results in high thermal stress in the coldest part of lithosphere. This simple case is not realistic because thermal stress can be high enough to cause tension cracking. (b) Thermal stress can be (at least partially) released by tension cracking. The release of thermal stress is limited by confining pressure, which is the difference between lithostatic and hydrostatic pressure, $(\rho-\rho_{\mathrm{fill}})gz$. Even if the complete release of thermal stress is somehow achieved [11], this brittle relaxation does not result in the recovery of fully equilibrated density, because the net density of fractured lithosphere is affected by crack porosity. This is compared with (hypothetical) complete viscous relaxation, which is shown in (c). See text for discussion.

response of oceanic lithosphere [11]. Because dislocation creep has higher activation energy, it quickly becomes inefficient at low temperature conditions. Note that at shallow depths, temperature is steadily decreasing because of half-space cooling. At any given depth, a time interval over which dislocation creep dominates diffusion creep is short (if it ever happens) because the cooling rate is fast enough to put the deformation regime back to diffusion creep. The analytical solution by [13] is thus sufficient to calculate $\alpha_{\text{eff}}^{\text{M}}/\alpha$.

Korenaga [11] also showed that, because of strongly temperature-dependent viscosity, viscous relaxation is severely retarded for the upper half of oceanic lithosphere where temperature is below ~700 °C. Accordingly, the accumulation of horizontal thermal stress is very efficient in shallow lithosphere, reaching the magnitude of a few hundred MPa or more. Such high thermal stress is expected to cause the brittle fracture of lithosphere by tension cracking, and this cracking releases thermal stress and allows more contraction.

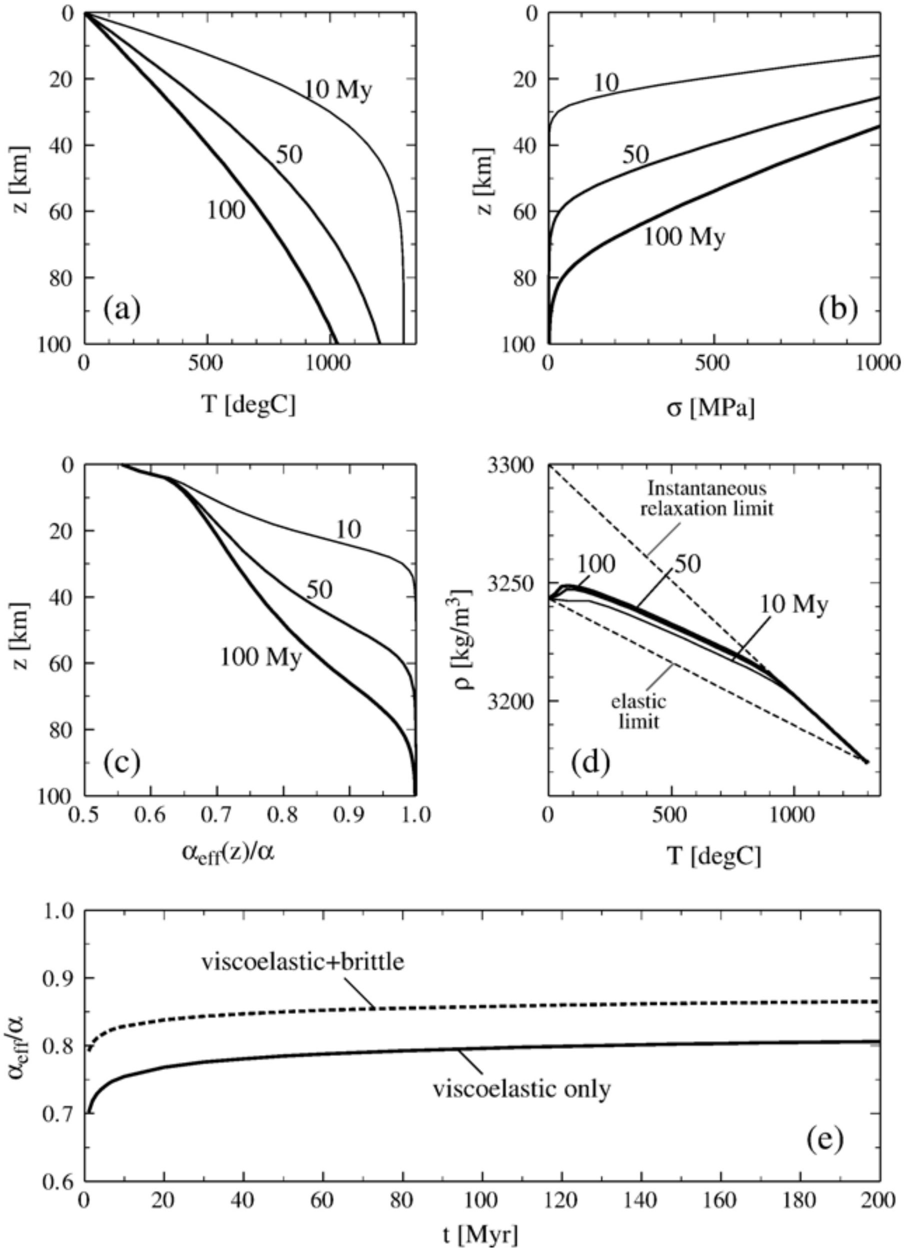

Fig. 3. Example of 1-D viscoelastic response with $\eta_0 = 10^{21}$ Pa s and $Q = 350$ kJ mol^{-1}. See text for other model parameters. (a) Temperature profiles after 10, 50, and 100 My of cooling. (b) Horizontal component of thermal stress. (c) Effective thermal expansivity calculated depth-wise and normalized by original thermal expansivity. (d) Relation between temperature and density. The instantaneous relaxation limit and the purely elastic limit are shown as dashed lines. (e) Effective thermal expansivity for viscoelastic relaxation (solid) and viscoelastic and complete brittle relaxation (dashed). Crack porosity is assumed to be filled by water for the latter case.

This process may be referred to as 'brittle' relaxation. One critical difference from viscous relaxation is that even when full thermal contraction (i.e., complete release of thermal stress) is achieved, brittle relaxation by tension cracking results in fractured lithosphere, the net density of which is lowered by crack porosity (Fig. 2). Furthermore, the complete release of thermal stress itself is not easy to attain because the difference between hydrostatic pressure and lithostatic pressure impedes crack growth. Crack growth modeling by [11] indicates that about half of thermal stress can be released by pervasive cracking in the presence of seawater, and also that an additional (currently unknown) mechanism is required to release the rest of residual thermal stress. Because of the confining pressure, vertical boundaries relevant to oceanic lithosphere (crack surfaces such as fracture zones and plate boundaries) are not guaranteed to be free-moving, and a realistic thermal stress state in the lithosphere can be well quantified by the following stepwise approach: (1) calculating thermal stress by the fixed vertical boundary condition and then (2) assessing the potential release of thermal stress by tension cracking.

Though the extent of brittle relaxation is somewhat uncertain as indicated above, a bound can still be obtained by assuming the complete release of thermal stress. Denoting the density of infilling material by ρ_{fill}, effective thermal expansivity after brittle relaxation should be bounded as follows:

$$\alpha_{\mathrm{eff}}^{\mathrm{M}} \le \alpha_{\mathrm{eff}} \le \alpha_{\mathrm{eff}}^{\mathrm{M}}\left(1-\frac{\rho_{\mathrm{fill}}}{\rho}\right) + \alpha\frac{\rho_{\mathrm{fill}}}{\rho}. \tag{11}$$

The most likely infilling material is, of course, seawater, and the introduction of water into cold suboceanic mantle may result in serpentinization. Note that the above bound with $\rho_{\mathrm{fill}}=\rho_{\mathrm{water}}$ is still valid even for the case of serpentinization because this chemical reaction hardly changes the total volume of mantle and infilling water as a whole.

3. A numerical example

To quantify the realistic range of viscoelastic relaxation in cooling oceanic lithosphere, the 1-D analytical model of [13] is used with the Arrhenius form of temperature-dependent viscosity:

$$\eta(T) = \eta_0 \exp\left(\frac{Q}{RT} - \frac{Q}{RT_0}\right), \tag{12}$$

where η_0 is reference viscosity at $T=T_0$, Q is activation energy, and R is universal gas constant. I test three reference viscosities, 10^{20}, 10^{21}, and 10^{22} Pa s, and activation energy is varied from 0 to 500 kJ mol^{-1}. Oceanic lithosphere is expected to be very dry because of dehydration upon melting beneath mid-ocean ridges [14,15], and the chosen range of reference viscosity is appropriate for this dry depleted state. A likely range of activation energy for diffusion creep is 300–400 kJ mol^{-1} [16]. Elastic parameters are set as: $\lambda=\mu=60$ GPa, which corresponds to the Young modulus of 150 GPa and the Poisson ratio of 0.25 [17]. Thermal expansivity is assumed to be 3×10^{-5} K^{-1}; note that the absolute value does not affect my calculation of $\alpha_{\mathrm{eff}}^{\mathrm{M}}/\alpha$. The half-space cooling is calculated with $T_{\mathrm{s}}=0$ °C, $T_{\mathrm{i}}=1300$ °C, and $\kappa=10^{-6}$ m^2 s^{-1}. The system depth is 300 km, and I calculate until $t=200$ My.

Fig. 3 shows the case of $\eta_0=10^{21}$ Pa s and $Q=350$ kJ mol^{-1}. The magnitude of thermal stress is on the order of a few hundred MPa, as a result of incomplete thermal contraction. For $T>\sim900$ °C, viscous relaxation is fast, resulting in $\alpha_{\mathrm{eff}}^{\mathrm{M}}/\alpha\sim1$ (Fig. 3c, d), and this ratio gradually approaches to the elastic limit as $T\rightarrow0$ °C. As viscous relaxation takes place everywhere with different time scales, its net effect for the entire lithosphere is time dependent (Fig. 3e). Except for the first 10 My or so, however, the temporal variation is minor, and effective thermal expansivity can be well approximated by a temporal average. For this particular case, effective expansivity is ~0.8 without brittle failure and ~0.85 with complete brittle relaxation (assuming

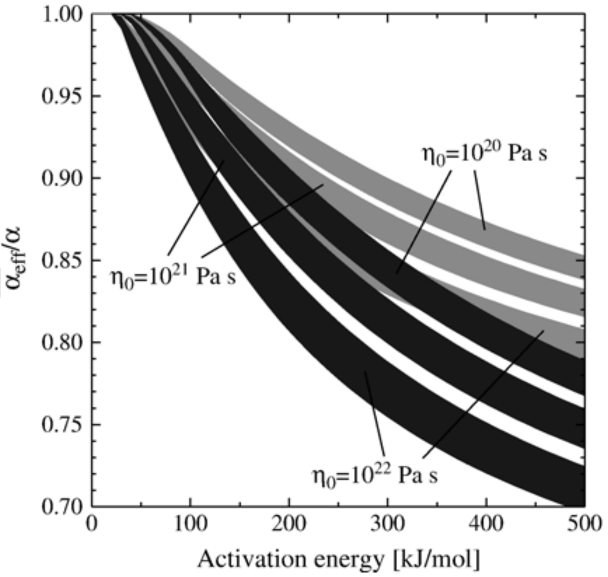

Fig. 4. Effective thermal expansivity averaged for 200 My, plotted as a function of activation energy. The range of $\pm1\sigma$ is shown. Solid shading is for purely viscoelastic response, and gray shading for complete brittle relaxation with seawater as infilling material.

$\rho_{fill} = \rho_{water}$). The temporal average of α_{eff}/α for all cases is summarized in Fig. 4.

Note that the viscosity of suboceanic mantle below ~70 km depth is likely to be lower than assumed here owing to the presence of dissolved hydrogen. However, cooling rate is substantially reduced at depths greater than 60–70 km, so viscous dissipation is efficient and thermal stress accumulation is already insignificant compared to shallower depths. If we assume complete thermal contraction is achieved at $z > 70$ km, it would result in <1% increase in effective thermal expansivity at $t = 100$ My and ~3% increase at $t = 200$ My for the case shown in Fig. 3. Thus, the effect of weak asthenosphere seems to be of second order.

4. Discussion and conclusion

The above calculations indicate that the viscoelastic effect can easily introduce up to ~15–30% reduction in effective thermal expansivity. Traditionally, the experimental determination of thermal expansivity tends to suffer from relatively large uncertainty (partly due to limited temperature range investigated), and this viscoelastic effect might be viewed as yet another source of uncertainty. However, more reliable data have been emerging in the last decade, and at least for pure forsterite, its thermal expansivity appears to be firmly established [5] (Fig. 5a); α is $\sim 3.1 \times 10^{-5}$ K^{-1} at room temperature and $\sim 4.0 \times 10^{-5}$ K^{-1} at 1300 °C. Of course, the mantle is not made of pure forsterite, and thermal expansivity for realistic mantle composition is still somewhat uncertain because we do not have all of necessary (high-quality) expansivity data. For oceanic lithosphere, the following two issues are important: (1) the effect of fayalite component for the expansivity of olivine, and (2) the thermal expansivity of orthopyroxene. The expansivity data of fayalite comparable to [5] does not exist at the moment, but by comparing old data for pure forsterite [18], Fo92 [19], and pure fayalite [20], all from the same laboratory (Fig. 5b), it may be reasonable to expect that the thermal expansivity of mantle olivine (Fo90–92 for oceanic lithosphere) would be different from that of pure forsterite by no more than a few percent. As clinopyroxene is preferentially consumed by melting, the second most abundant mineral in oceanic lithosphere is orthopyroxene [21]; olivine and orthopyroxene together constitute about 90% of depleted lithosphere. Just like the case for olivine, the expansivity data for orthopyroxene have been controversial, but the most recent experiment [22] indicates that olivine and orthopyroxene may actually share a similar thermal expansivity (Fig. 5b). As far as mineral physics is concerned, therefore, the thermal expansivity of oceanic lithosphere is now reasonably well understood and could be approximated as that of pure forsterite.

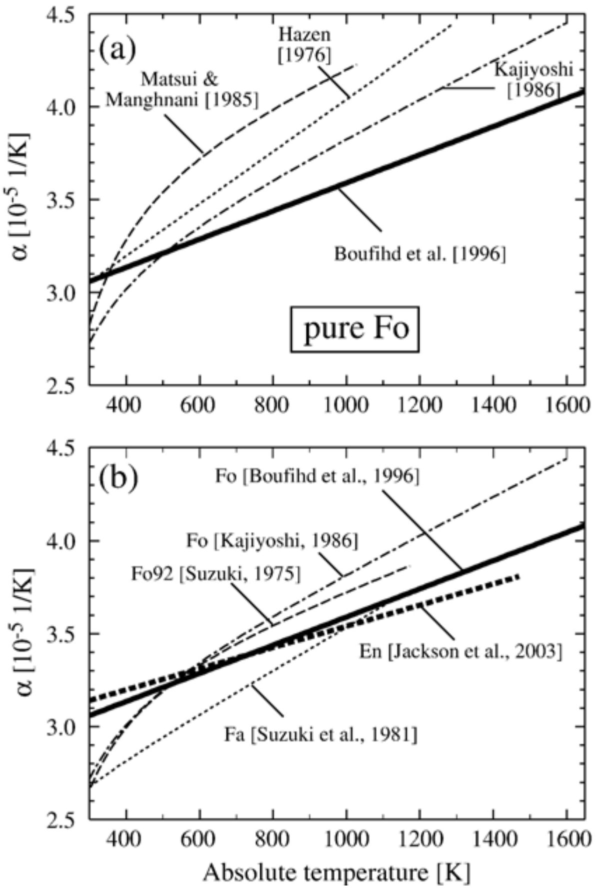

Fig. 5. (a) Summary of mineral physics data for the thermal expansivity of pure forsterite: Hazen [24] (dotted), Matsui and Manghnani [25] (dashed), and Kajiyoshi [18] (dot-dashed), and Boufihd et al. [5] (thick solid). The experiment by Boufihd et al. [5] goes up to 2100 K. (b) Thermal expansivity data relevant to oceanic lithosphere: pure forsterite by [5] (thick solid) and natural orthoenstatite, $(Mg_{0.994}Fe_{0.002}Al_{0.004})_2(Si_{0.996}Al_{0.004})_2O_6$, by [22] (thick dotted), together with old olivine data from Suzuki's lab, Fo92 [19] (dashed), fayalite [20] (dotted), and forsterite [18] (dot-dashed).

The effect of viscous relaxation suggested by this study is thus not trivial, and more importantly, it *systematically* lowers the apparent thermal expansivity. It is interesting to note that α of 3×10^{-5} K^{-1} has long been a standard value for the upper mantle in geophysical studies [17], even though in the past three decades mineral physics data for forsterite have persistently indicated this value is only appropriate near the room temperature condition. The long-standing preference to 3×10^{-5} K^{-1} may stem from the analysis of seafloor subsidence. Though their estimates are subject to large uncertainty, Parsons and Sclater [7] and Stein and Stein [8] both show that this value is nonetheless the best-fit value given the observed seafloor subsidence. Incomplete thermal contraction due to temperature-dependent viscosity may explain this low expansivity.

To sum, the effective thermal expansivity of oceanic lithosphere is a function of mantle rheology. Because the effect of viscoelastic response is shown to be nearly time-independent, it can be concisely represented by a single time-averaged reduction factor $\overline{\alpha}_{\mathrm{eff}}/\alpha$. A bound for possible brittle relaxation can also be calculated by assuming the density of infilling material. The simplicity of this result should allow us to easily simulate effective expansivity in the calculation of heat conduction and corresponding subsidence. It will be shown elsewhere [23] that the concept of effective thermal expansivity is essential to understand the origin of depth anomalies at old ocean floor.

Acknowledgments

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References

[1] M.P. Doin, L. Fleitout, Thermal evolution of the oceanic lithosphere: an alternative view, Earth Planet. Sci. Lett. 142 (1996) 121–136.

[2] A.M. Hofmeister, Mantle values of thermal conductivity and the geotherm from phonon lifetimes, Science 283 (1999) 1699–1706.

[3] S. Honda, D.A. Yuen, Interplay of variable thermal conductivity and thermal expansivity on the thermal structure of oceanic lithosphere, Geophys. Res. Lett. 28 (2001) 351–354.

[4] D. McKenzie, J. Jackson, K. Priestley, Thermal structure of oceanic and continental lithosphere, Earth Planet. Sci. Lett. 233 (2005) 337–349.

[5] M.A. Bouhifd, D. Andrault, G. Fiquet, P. Richet, Thermal expansion of forsterite up to the melting point, Geophys. Res. Lett. 23 (1996) 1143–1146.

[6] R.G. Berman, L.Y. Aranovich, Optimized standard state and solution properties of minerals: 1. model calibration for olivine, orthopyroxene, cordierite, garnet, and ilmenite in the system FeO–MgO–CaO–Al_2O_3–TiO_2–SiO_2, Contrib. Mineral. Petrol. 126 (1996) 1–24.

[7] B. Parsons, J.G. Sclater, An analysis of the variation of ocean floor bathymetry and heat flow with age, J. Geophys. Res. 82 (1977) 803–827.

[8] C.A. Stein, S. Stein, A model for the global variation in oceanic depth and heat flow with lithospheric age, Nature 359 (1992) 123–129.

[9] D.L. Kohlstedt, B. Evans, S.J. Mackwell, Strength of the lithosphere: constraints imposed by laboratory experience, J. Geophys. Res. 100 (1995) 17587–17602.

[10] H.S. Carslaw, J.C. Jaeger, Conduction of Heat in Solids, 2nd ed. Oxford, New York, 1959.

[11] J. Korenaga, Thermal cracking and the deep hydration of oceanic lithosphere: A key to the generation of plate tectonics? J. Geophys. Res. (in press).

[12] B.A. Boley, J.H. Weiner, Theory of Thermal Stresses, John Wiley & Sons, New York, 1960.

[13] R. Muki, E. Sternberg, Transient thermal stresses in visco-elastic materials with temperature dependent properties, J. Appl. Mech. 28 (1961) 193–207.

[14] S. Karato, Does partial melting reduce the creep strength of the upper mantle? Nature 319 (1986) 309–310.

[15] G. Hirth, D.L. Kohlstedt, Water in the oceanic mantle: implications for rheology, melt extraction, and the evolution of the lithosphere, Earth Planet. Sci. Lett. 144 (1996) 93–108.

[16] G. Hirth, D. Kohlstedt, Rheology of the upper mantle and the mantle wedge: a view from the experimentalists, in: J. Eiler (Ed.), Inside the Subduction Factory, American Geophysical Union, Washington, D.C., 2003, pp. 83–105.

[17] D.L. Turcotte, G. Schubert, Geodynamics: Applications of Continuum Physics to Geological Problems, John Wiley & Sons, New York, 1982.

[18] K. Kajiyoshi, High temperature equations of state for mantle minerals and their anharmonic properties. Master's thesis, Okayama Univ., Okayama, Japan, 1986.

[19] I. Suzuki, Thermal expansion of periclase and olivine and their anharmonic properties, J. Phys. Earth 23 (1975) 145–159.

[20] I. Suzuki, K. Seya, H. Takei, Y. Sumino, Thermal expansion of fayalite, Fe_2SiO_4, Phys. Chem. Miner. 7 (1981) 60–63.

[21] R.J. Kinzler, T.L. Grove, Primary magmas of mid-ocean ridge basalts, 2, applications, J. Geophys. Res. 97 (1992) 6907–6926.

[22] J.M. Jackson, J.W. Palko, D. Andrault, S.V. Sinogeikin, D.L. Lakshtanov, J. Wang, J.D. Bass, C.-S. Zha, Thermal expansion of natural orthoenstatite to 1473 K, Eur. J. Mineral. 15 (2003) 469–473.

[23] T. Korenaga, J. Korenaga. The origin of seafloor 'flattening' revealed by residual statistics. Earth Planet. Sci. Lett. (in revision).

[24] R.M. Hazen, Effects of temperature and pressure on the crystal structure of forsterite, Am. Mineral. 61 (1976) 1280–1293.

[25] T. Matsui, M.H. Manghnani, Thermal expansion of single-crystal forsterite to 1023 K by Fizeau interferometry, Phys. Chem. Miner. 12 (1985) 201–210.