Thermal evolution of Earth with magnesium precipitation in the core

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A B S T R A C T

Vigorous convection in Earth’s core powers our global magnetic field, which has survived for over three billion years. In this study, we calculate the rate of entropy production available to drive the dynamo throughout geologic time using one-dimensional parameterizations of the evolution of Earth’s core and mantle. To prevent a thermal catastrophe in models with realistic Urey ratios, we avoid the conventional scaling for plate tectonics in favor of one featuring reduced convective vigor for hotter mantle. We present multiple simulations that capture the effects of uncertainties in key parameters like the rheology of the lower mantle and the overall thermal budget. Simple scaling laws imply that the heat flow across the core/mantle boundary was elevated by less than a factor of two in the past relative to the present. Another process like the precipitation of magnesium-bearing minerals is therefore required to sustain convection prior to the nucleation of the inner core roughly one billion years ago, especially given the recent, upward revision to the thermal conductivity of the core. Simulations that include precipitation lack a dramatic increase in entropy production associated with the formation of the inner core, complicating attempts to determine its age using paleomagnetic measurements of field intensity. Because mantle dynamics impose strict limits on the amount of heat extracted from the core, we find that the addition of radioactive isotopes like potassium-40 implies less entropy production today and in the past. On terrestrial planets like Venus with more sluggish mantle convection, even precipitation of elements like magnesium may not sustain a dynamo if cooling rates are too slow.

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1. Introduction

The dynamo created in Earth’s liquid outer core has survived for billions of years. Paleomagnetic studies of unmetamorphosed rocks with ages near 3.45 Gyr unambiguously show that the strength of Earth’s global magnetic field at that time was at least half its present-day value (e.g., Tarduno et al., 2010; Biggin et al., 2011). No rocks of sufficiently low metamorphic grade have been found from earlier epochs, so the question of whether our magnetic field is even older remains unanswered. Recently, detrital zircon crystals found in the Jack Hills of Western Australia were proposed to record field intensities of modern magnitudes (Tarduno et al., 2015). These data are controversial, however, because zircon-bearing rocks in the Jack Hills may have suffered pervasive re-magnetization related to the emplacement of a nearby igneous province (e.g., Weiss et al., 2015). In any case, how to power convection in the core and thus a dynamo for the vast majority of Earth’s history remains one of the most pressing puzzles in geophysics.

Thermal convection in the core is possible if the heat flow across the core/mantle boundary (CMB) exceeds the rate at which heat is conducted along an adiabatic temperature gradient (e.g., Stevenson, 2003). Over the past few years, some theoretical calculations (e.g., de Koker et al., 2012; Pozzo et al., 2012) and diamond-anvil cell experiments (e.g., Gomi et al., 2013; Seagle et al., 2013; Ohta et al., 2016) have indicated that the thermal conductivity of the core’s iron-rich alloy is a factor of two to three larger than prior estimates. The conductive heat flux is ~10–15 TW at present according to these new values. However, countervailing evidence from high-pressure experiments that the previous, low values are actually correct has also been presented recently, so debate over this issue will likely continue (Konôpková et al., 2016).

Cooling rates approaching twice the conductive heat flux have been suggested as the minimum required to compensate for Ohmic dissipation (e.g., Stelzer and Jackson, 2013). But this dissipation mainly occurs at high harmonic degree and its scaling with dipole field strength is uncertain. Since the dissipation due to the low harmonics alone is far less than the actual heat flow, maintaining the observed field with a heat flow only mildly in excess of conduction along the adiabat is possible in principle. In any case, the actual CMB heat flow of ~5–15 TW estimated from seis-
ology and mineral physics (e.g., Lay et al., 2008) may be only marginally sufficient to sustain the dynamo by thermal convection alone. Fortunately, the dynamic chemistry of the core yields additional sources of energy.

The exclusion of light elements from the solidifying inner core provides enough compositional buoyancy to drive convection today. Once compositional buoyancy is present, the heat flow out of the core need not exceed conduction along the adiabat (i.e., convection can even carry heat downwards). In practice, models with a growing inner core also benefit from the significant release of latent heat and accordingly require less rapid cooling. Conventional calculations have indicated that the inner core nucleated roughly one billion years ago (e.g., Labrosse et al., 2001). The age of the inner core is several hundred million years less in models with increased CMB heat flow and thus faster cooling/freezing to accommodate the revised values for thermal conductivity (e.g., Nimmo, 2015; Labrosse, 2015).

The energy available for dissipation in dynamo generation dramatically increases once the inner core forms, which might imply a larger magnetic field according to scaling laws where the buoyancy flux determines the global field strength (e.g., Christensen, 2010). In some canonical models, the inner core thus prevents the dynamo from turning off (e.g., Stevenson et al., 1983), but these models do not explain the current total heat flow of Earth. Biggin et al. (2015) claimed to observe an increase in Earth’s dipole moment associated with the formation of the inner core in the Mesoproterozoic. Given the relevant experimental and statistical uncertainties, however, the available data are arguably consistent with roughly constant field intensities throughout the Precambrian (e.g., Smirnov et al., 2016).

O’Rourke and Stevenson (2016) proposed the precipitation of magnesium-bearing minerals as an alternative power source. One or two weight percent of magnesium can partition into the core in the high-temperature aftermath of giant impacts during Earth’s accretion according to earlier calculations (Wahl and Militzer, 2015) and subsequent diamond-anvil cell experiments (Badro et al., 2016). Because its solubility in iron alloy is strongly-temperature dependent, subsequent cooling quickly saturates the core in magnesium. Elements like aluminum and calcium may have similar thermodynamic properties (Badro et al., 2016), but their abundances are relatively small. Transporting magnesium-rich oxide or silicate across the CMB provides an order-of-magnitude more gravitational energy than freezing an equivalent mass of the inner core. Precipitation drives vigorous, compositional convection before the nucleation of the inner core, even without vastly higher CMB heat flow than today. O’Rourke and Stevenson (2016), however, only calculated the CMB heat flow implied by a constant rate of entropy production for the dynamo. In reality, mantle dynamics control CMB heat flow, so entropy production should vary over time.

The purpose of this paper is to describe simple models of Earth’s thermal evolution that are consistent with the observed longevity of the dynamo. First, we describe how we couple a one-dimensional model of the core to simple scaling laws for mantle dynamics. We next identify which parameters control the amount of power available for the dynamo throughout geologic time. Specifically, we focus on the rheology of the boundary layer at the base of the mantle and the abundance of radioactive isotopes like potassium–40 in the core. After presenting representative simulations, we discuss the limitations of our model for early Earth history and the implications for other planets.

2. Theoretical formulation

In this section, we present a parametrized model for the coupled evolution of Earth’s core and mantle. Fig. 1 shows the simplified structure with which we calculate thermal histories. Key model parameters are listed in Table 1. As in nearly all models of core history for the past fifty years, we assume that the core is sufficiently low viscosity that the convective state is extremely close to an isotropic and homogeneous state, except in thin boundary layers (e.g., Stevenson, 1987). Although most previous studies (e.g., Stevenson et al., 1983; Buffett, 2002) only consider a thermal boundary layer at the base of the mantle, we allow for the existence of a stagnant layer that may not participate in convection because it is compositionally dense (Hernlund and McNamara, 2015), possibly the solidified remnant of a basal magma ocean (e.g., Labrosse et al., 2007). The existence of this distinct chemical layer could explain why the thermal excess associated with mantle plumes may be less than half the total temperature contrast across the CMB (e.g., Farnetani, 1997). Because our primary focus is how mantle dynamics affect the evolution of the core, we do not model the dynamics of the crust and lithosphere in detail. Finally, we present simulations that demonstrate the effects of varying key parameters.

![Fig. 1. Cartoon showing the assumed thermal structure of Earth and the key parameters tracked during simulations of Earth’s evolution. The temperature gradients and vertical dimensions of each layer are not to scale.](image-url)
2.1. Evolution of the mantle

The governing equation here is the global energy balance for the mantle (e.g., Christensen, 1985):

\[ C_M \frac{d T_M(t)}{d t} = H_M(t) - Q_M(t) + Q_{CMB}(t), \]

where \( C_M \) is the heat capacity of the entire mantle, \( T_M \) is the potential temperature of the mantle, \( H_M \) is the radiogenic heating in the convecting mantle, \( Q_M \) is the heat flow out of the surface from mantle convection, and \( Q_{CMB} \) is heat flow across the core/mantle boundary. Heat-producing elements are partially sequestered in the continental crust, so \( H_M \) is less than the present-day heat production of the bulk silicate Earth. Table 2 lists the values we adopt for parameters like \( C_M \) that are generally fixed in our simulations.

At present, how to calculate \( Q_M \) remains quite controversial. Conventional scalings for \( Q_M \) assume that the mantle behaves like a simple convecting system in which hotter temperature yields an increased vigor of convection and thus heat flow. But there is a well-known problem with these scalings that arises when the convective Urey ratio, \( U_M(t) = H_M(t)/Q_M(t) \), is considered (Christensen, 1985). Integrating Eq. (1) backwards in time yields unrealistically high values of \( T_M \) unless the present-day \( U_M(t_0) \sim 0.75 \). Robust geochemical constraints, however, imply that \( U_M(t_0) \sim 0.2 \) to 0.4 in reality (e.g., Korenaga, 2008; Lay et al., 2008).

Using the realistic Urey ratio in simulations of Earth’s evolution produces a “thermal catastrophe” in which calculated mantle temperatures before \( \sim 2–3 \) Ga rapidly exceed petrological constraints (e.g., Herzberg et al., 2010). To avoid this problem, we calculate \( Q_M \) as a function of \( T_M \) following Korenaga (2006). This simple formulation is sufficient for the purposes of this study and is consistent with fully dynamical models (e.g., Korenaga, 2010), especially considering uncertainties related to mantle hydration and the scaling of bending dissipation for subducting plates (e.g., Rose and Korenaga, 2011; Korenaga, 2011).

The properties of the thermal boundary layer at the base of the mantle determine the core/mantle heat flow. We incorporate a simple boundary-layer model resting on the assumption that convective instability occurs once a local Rayleigh number reaches a critical value. Thus, we calculate the value of \( Q_{CMB} \) at any epoch relative to the present (Buffett, 2002):

\[ Q_{CMB}(t_1) = \frac{(T_U(t_1) - T_B(t_1))^4}{(T_U(t_0) - T_B(t_0))^4} \left( \frac{\eta_B(T(t_0))}{\eta_B(T(t_1))} \right)^{\frac{1}{4}}, \]

where \( T(t) = \frac{(T_U(t) + T_B(t))/2 \text{ is the average temperature in the thermal boundary layer. Assuming whole-mantle convection as indicated by seismic tomography of slabs and plumes (e.g., van der Hielt et al., 1997; French and Romanowicz, 2015), the temperature at the base of the mantle scales as } T_B(t_j) = \frac{T_M(t_j)/T_M(t_j)}{T_B(t_j)}. \] If convection in the mantle were layered, then \( T_B \) would increase by the temperature contrast across the mid-mantle transition layer. Hence, the estimated temperature contrast across the CMB would decrease.

The effective viscosity of the thermal boundary layer is calculated as (Korenaga, 2005):

\[ \eta_B(T) = \eta_B(T_{ref}) \exp \left[ \frac{H_{eff}}{R T B} - \frac{H_{eff}}{R T_{ref}} \right], \]

where \( H_{eff} \) is the activation enthalpy, \( R \) is the universal gas constant, and \( \eta_B(T_{ref}) \) is the reference viscosity, comparable to the average viscosity of the lower mantle, at some reference temperature \( T_{ref} \). Typical values assumed for \( H_{eff} \) are on the order of \( \sim 300 \text{ kJ mol}^{-1} \). But the rheology of the lower mantle is uncertain enough that negative values are also possible, in which case hotter material would have higher viscosity (e.g., Solomatov, 1996; Korenaga, 2005).

Assuming the stagnant layer is in a steady state, the temperature at the top of the stagnant layer is easily calculated (e.g., Turcotte and Schubert, 2002):

\[ T_U = T_{CMB} - \frac{Q_{CMB}}{4 \pi k_M} \left[ \frac{1}{R_C} - \frac{1}{R_S} \right], \]

where \( T_{CMB} \) is the temperature at the top of the core, \( k_M \) is the thermal conductivity of the lower mantle, and \( R_S = R_C + d_S \) is the distance from the center of Earth to the top of the stagnant layer. The effective thickness of the stagnant layer is \( d_S \). Small changes to \( d_S (\sim 10 \text{ km}) \) are degenerate with the slight decrease in \( T_U \) caused by plausible rates of radiogenic heating in the stagnant layer, which we neglect.

2.2. Energetics of the core

The global energy balance for the core is (e.g., Labrosse, 2015):

\[ Q_{CMB} = Q_R + Q_s + Q_P + Q_C + Q_L, \]

where \( Q_R \) is radiogenic heating and \( Q_s \) is secular cooling. Gravitational energy associated with the precipitation of magnesium-bearing minerals is \( Q_P \) (e.g., Buffett et al., 2000; O’Rourke and Stevenson, 2016). The final two terms are the gravitational energy (\( Q_C \)) and latent heat (\( Q_L \)) associated with the growth of the inner core. Note that the ohmic dissipation of the dynamo is not included here, because such heating is both generated and dissipated entirely within the core. Analytic expressions for all but one term are available in Labrosse (2015), along with associated constants like the density contrast at the inner core boundary and the slopes of the liquidus and isentropic temperature gradients. We derive a polynomial expression for \( Q_P \) in the Appendix. In O’Rourke and Stevenson (2016), we presented an expression for \( Q_s + \) compatible with the formulation of Nimmo (2015). We use the fourth-order expansion of Labrosse (2015) in this paper for a better match to the density structure of the core from PREM and estimates of the heat gradients at the top of the core.
where \( r \) is radial distance, \( L_p \) is derived from the equation of state for the liquid core alloy, and \( A_k \) is a constant. According to most recent studies, the thermal conductivity at the center of the core is \( k_C(0) \approx 163 \text{ W m}^{-1} \text{ K}^{-1} \) (e.g., Labrosse, 2015). But we run some simulations using values as low as 40 \text{ W m}^{-1} \text{ K}^{-1} (Konopkova et al., 2016).

Each of the terms besides \( Q_R \) are proportional to the cooling rate of the core. That is,

\[
Q_{CMB} = Q_R + \left( \tilde{Q}_S + \tilde{Q}_P + \tilde{Q}_C + \tilde{Q}_L \right) \frac{dT_{CMB}}{dt},
\]

where \( \tilde{Q}_i = Q_i/(dT_{CMB}/dt) \). The growth rate of the inner core is simply proportional to the overall cooling rate as \( dR_i/\rho_i = \gamma_i (dT_{CMB}/dt) \). Here, we use a conversion factor (Nimmo, 2015):

\[
\gamma_i = -\frac{1}{dT_m/dP - dT_a/dP} \left( \frac{T_i}{\rho_i T_{CMB}} \right),
\]

where \( dT_m/dP \) and \( dT_a/dP \) are the slopes of the melting curve and adiabatic temperature gradient, respectively, at the inner core boundary. Likewise, \( T_R, T_S, \) and \( T_L \) are the temperature and density at the inner core boundary calculated from the adiabatic profiles in Labrosse (2015).

Another equation expresses the conservation of entropy production (e.g., Gubbins, 1977; Labrosse, 2015):

\[
\frac{Q_{CMB}}{T_{CMB}} = \frac{Q_R}{T_R} + \frac{Q_S}{T_S} + \frac{Q_L}{T_L} + E_K + E_\phi,
\]

where \( T_R, T_S, \) and \( T_L \) are effective temperatures at which the respective heat sources are dissipated. The entropy production rates associated with conductive heat transport along the adiabatic temperature gradient and ohmic dissipation are \( E_K \) and \( E_\phi \), respectively.

Rearranging Eq. (7), the cooling rate of the core is

\[
\frac{dT_{CMB}}{dt} = \frac{Q_{CMB} - Q_R}{Q_S + Q_P + Q_C + Q_L}.
\]

Combining Eqs. (7) and (9), we finally calculate the entropy available to sustain the dynamo

\[
E_\phi = \frac{Q_{CMB}}{T_{CMB}} - \frac{Q_R}{T_R} - \left( \frac{Q_S}{T_S} + \frac{Q_L}{T_L} \right) \frac{dT_{CMB}}{dt} - E_K.
\]

To sustain a dynamo, \( E_\phi \) must of course be positive. Something in the rather wide range of \( \sim 20\text{–}500 \text{ MW K}^{-1} \) is probably required, and the actual minimum value is poorly constrained because ohmic dissipation occurs at short length scales that are difficult to simulate (e.g., Gubbins, 1977). We can estimate the energy associated with ohmic dissipation as \( Q_\phi = T_\phi E_\phi \), where \( T_\phi \approx 5000 \text{ K} \) is some characteristic temperature of dissipation between \( T_{CMB} \) and the temperature at the inner core boundary (Nimmo, 2015). The dissipation rate is an acceptable proxy for magnetic field strength, although more complicated scaling laws have been formulated (e.g., Christensen, 2010).

Only a portion of core formation occurred in the aftermath of giant impacts. Since the equilibration temperature for most material was \( \sim 4500 \text{ K} \), the core was likely undersaturated in light elements at first, assuming full mixing and an initially homogeneous core. Thus, precipitation of magnesium-bearing minerals was delayed until after an initial episode of cooling (O’Rourke and Stevenson, 2016; Badro et al., 2016). Once started, however, precipitation continues until the core is entirely depleted in light elements. Here we assume that precipitation has been occurring for the entire length of our simulations, meaning that the core became saturated within \( \sim 500 \text{ Myr} \) after accretion.

Based on diamond-anvil cell experiments conducted at extreme temperature/pressure conditions, Badro et al. (2016) determined that pure MgO would precipitate at a rate \( C_M \approx 2.5 \times 10^{-5} \text{ K}^{-1} \) normalized to the total mass of the core. O’Rourke and Stevenson (2016) included SiO\(_2\) and FeO in the precipitate and did not assume the core and mantle were in equilibrium after accretion, yielding a larger \( C_M \approx 5 \times 10^{-5} \text{ K}^{-1} \). These calculations were based on extrapolations of earlier experiments conducted at lower temperatures, but Badro et al. (2016) obtained roughly consistent expressions for the relevant exchange coefficients. En-tropic arguments suggest that the precipitate should include every element present in the outer core. Because Mg is least soluble, the precipitate is initially MgO-rich but contains increasing amounts of SiO\(_2\) as \( T_{CMB} \) decreases. The initial abundances of each element—several combinations of which satisfy constraints from seismology and mineral physics (e.g., Badro et al., 2014; Fischer et al., 2015)—dictate the evolving composition of the precipitate. Given the myriad uncertainties, we use the intermediate value \( C_M = 4 \times 10^{-5} \text{ K}^{-1} \) for most simulations but also describe the implications of higher or lower values.

If \( E_\phi \) is assumed to have been roughly constant throughout geologic time, then we can modify the above equations to calculate the implied values of \( Q_{CMB} \) and \( T_{CMB} \) in the past (e.g., O’Rourke and Stevenson, 2016). However, since mantle dynamics actually control \( Q_{CMB} \) and thus \( T_{CMB} \) as detailed above, using Eq. (11) and a coupled model of core/mantle evolution is required to determine what scenarios are compatible with the observed longevity of Earth’s dynamo.

2.3. Calculating thermochemical histories

Various observational constraints on the thermal budget of Earth today are available. The total heat flux at the surface is \( 44 \pm 3 \text{ TW} \) (Jaupart et al., 2007). Estimates of the present-day heat production in the bulk silicate Earth range from \( 16 \pm 3 \text{ TW} \) (Lyubetskaya and Korenaga, 2007) to \( \sim 20 \text{ TW} \) (Jaupart et al., 2007). Arguments from mineral physics and seismology have implied that the core/mantle boundary heat flow is currently \( \sim 6 \) to 15 TW (e.g., Lay et al., 2008). With heat production in the continental crust estimated as \( \sim 6 \) to 8 TW (Jaupart et al., 2007), radiogenic heating in the mantle is perhaps \( 6 \) to 14 TW and reasonable values for the mantle heat flux might be \( \sim 33 \) to 41 TW. Experiments on metal-silicate partitioning suggest that the abundance of potassium in the core is less than 200 ppm (e.g., Corgne et al., 2007), implying that radiogenic heating in the core is \( \sim 0.1 \text{ TW} \) at present.

Absolute temperatures within Earth now are comparably uncertain. Extrapolating temperatures of the relevant phase transitions at the mantle’s transition zone down an adiabatic gradient imply that the basal temperature of the mantle is \( \sim 2500 \) to 2800 K. With a present-day temperature of \( \sim 4000 \text{ K} \) at the top of the core (Labrosse, 2015), the temperature contrast across the core/mantle boundary is \( \sim 1000 \) to 1800 K, much larger than the thermal excess of \( \sim 500 \text{ K} \) attributed to mantle plumes (e.g., French and Romanowicz, 2015). Note that this thermal excess may diminish by a factor of roughly two as plumes ascend from the CMB to the upper mantle. That is, a near-surface thermal excess of \( \sim 250 \text{ K} \) may imply a temperature difference of \( \sim 500 \text{ K} \) across the thermal boundary layer at the base of the mantle (e.g., Leng and Zhong, 2008).

Using the present as an “initial” condition, we can integrate the equations presented above backwards in time to calculate a thermochemical history of Earth. The present-day thermal budget has significant uncertainties, but constraints on the state of the mantle and core at the time that plate tectonics began are obvi-
ously much weaker. This procedure may not reproduce the state of the core and mantle throughout the Hadean because scaling laws other than those presented above or more complicated numerical simulations are required to model the aftermath of giant impacts, the solidification of the primordial magma ocean, and any regime of mantle dynamics that may have preceded plate tectonics. Accordingly, the primary utility of our approach is to reconstruct a thermal history for the mantle consistent with geologic evidence for the period when the geodynamo definitely existed.

In every simulation, we assume that $T_B(t_0) = 2800$ K and that continents grew to their modern size by 4 Ga. Using the adiabatic temperature gradient for the core from Labrosse (2015) implies that $T_{CMB} \approx 4050$ K given the present-day radius of the inner core, $R_I = 1220$ km. Unless otherwise indicated, we use the following set of “nominal” parameters: $Q_M(t_0) = 36$ TW, $Q_{CMB}(t_0) = 10$ TW, $H_M(t_0) = 10$ TW, $H_{eff} = 300$ kJ mol$^{-1}$ and $|K| = 50$ ppm in the core. With 8 TW of radiogenic heating in the continental crust in this case, the present-day heat flux thus totals 44 TW. We also assume that the effective thickness of the stagnant layer, $d_s = 50$ km, except when $Q_{CMB}(t_0)$ is varied.

3. Results

Fig. 2 shows the results of the thermal evolution simulation using our nominal parameters. With the rate of magnesium precipitation $C_M = 4 \times 10^{-5}$ K$^{-1}$ (normalized to the total mass of the core), $E_P > 500$ MW K$^{-1}$ at all times. The entropy production rises to maxima of $\sim 670$ and $630$ MW K$^{-1}$ near $0.65$ and $2.5$ Ga, respectively, equivalent to ohmic dissipation rates of $Q_\phi \approx 3.1-3.4$ TW that are well above the minimum estimated to sustain a dynamo (e.g., Nimmo, 2015). The age of the inner core is $\sim 0.83$ Ga, at which point $Q_L$ and $Q_S$ disappear and the entropy production rate reaches a local minimum. At present day, the temperature differences across the thermal boundary layer and the adjacent stagnant layer are both $\sim 600$ K, which roughly matches constraints on the thermal excess associated with mantle plumes and the total temperature contrast across the core/mantle boundary. For the entire simulation, the change in entropy content associated with thermal conduction is as large as the total entropy production available for the dynamo (i.e., $E_K \approx E_\phi$). Precipitation is critical to the operation of a dynamo before inner core nucleation, even though $Q_P \sim 0.5Q_S$. That is, the contribution of secular cooling to the total dissipation is penalized by a Carnot-like efficiency term $\sim (T_S - T_{CMB})/T_{CMB}$ relative to compositional buoyancy from precipitation or the inner core (Nimmo, 2015; Labrosse, 2015).

Fig. 3 illustrates the effects of varying the rate of magnesium precipitation. Five simulations were performed with $C_M$ increasing from $0$ to $8 \times 10^{-5}$ K$^{-1}$ in increments of $2 \times 10^{-5}$ K$^{-1}$. Increasing $C_M$ yields increased entropy production rates, along with decreased $Q_{CMB}$ and $T_{CMB}$ in the past. At least some precipitation is required to maintain positive values of $E_P$ before nucleation of the inner core. Moreover, values of $C_M \geq 4 \times 10^{-5}$ K$^{-1}$ are preferred because $E_\phi$ must be significantly larger than zero to sustain a global magnetic field (e.g., Nimmo, 2015). Magnesium precipitation notably limits the extent to which $E_\phi$, and thus presumably the strength of Earth’s magnetic field recorded at the surface, reaches a local minimum at the time of inner core nucleation. That is, entropy production rates are roughly constant within $\sim 10\%$ throughout geologic time in simulations with $C_M \geq 4 \times 10^{-5}$ K$^{-1}$.

We repeated the simulations shown in Fig. 3 with the thermal conductivity decreased from $k_C(0) = 163$ to 40 W m$^{-1}$ K$^{-1}$. With all other parameters held constant, $E_K$ is the only term affected. Thus, the evolution of $Q_{CMB}$ and $T_{CMB}$ is unchanged, while $E_\phi$ is increased by $\sim 400$ MW K$^{-1}$ at all times. If the lowest estimates of thermal conductivity are actually correct (Konôpková et al., 2016), then magnesium precipitation is not required to maintain positive dissipation. However, at least $C_M = 2 \times 10^{-5}$ is still necessary to unambiguously sustain a dynamo with $E_\phi > 500$ MW K$^{-1}$ at all times.

Fig. 4 elucidates how varying the mantle heat flow affects the evolution of the core. Simulations were performed with $Q_M(t_0)$...
varied from 32 to 40 TW to represent the uncertainties in the thermal budget of Earth. In each of these simulations, $C_M = 4 \times 10^{-5} \text{ K}^{-1}$, $H_M(t_0) = 10 \text{ TW}$, and 8 TW of radiogenic heating is assumed for the continental crust. Increasing $Q_M(t_0)$ implies more entropy production for the dynamo in the past, along with higher values of both $Q_M$ and $T_M$. The effect on the dynamics of the core, however, is relatively small compared to the uncertainties centered on the other parameters described above. In these simulations, the present-day Urey ratio is $\sim 0.3$, increasing to $\sim 1.2$ at 4 Ga. However, using a high present-day Urey ratio ($\sim 0.75$), together with conventional scaling laws for mantle dynamics, would only marginally affect our results, at least during the few billion years before the “thermal catastrophe” renders the mantle globally molten.

**Fig. 3.** Multiple simulations showing that increasing the precipitation rate of magnesium-bearing minerals increases the entropy production for the dynamo (a) and, relative to the present, decreases the implied temperatures of the core (b), core/mantle heat flow (c) in the past.

is difficult, requiring $H_{eff} \geq 600 \text{ kJ mol}^{-1}$ and the absence of any compositionally-distinct, stagnant layer.

The rheology of the lower mantle is poorly constrained. If the grain size-dependent part of diffusion creep dominates, then hotter mantle may actually have higher viscosity (Solomatov, 1996; Korenaga, 2005). In simulations with negative values of $H_{eff}$, hotter temperatures in the past would imply a thicker thermal boundary layer at the base of the mantle, leading to inhibited $Q_{CMB}$ and thus a lower likelihood of sustaining a dynamo. With $H_{eff} = -300 \text{ kJ mol}^{-1}$, the viscosity contrast across the thermal boundary layer is roughly one order of magnitude as long as a stagnant, compositionally-distinct layer with $d_s = 50 \text{ km}$ is still present. If $H_{eff}$ were even more negative, however, then the viscosity contrast may become large enough that the bottom portion of the thermal boundary layer would itself stagnate, despite its compositional homogeneity (e.g., Solomatov and Moresi, 2000). In this case, calculating $Q_{CMB}$ is more complicated (Korenaga, 2005), and a compositionally distinct layer is not required to explain the different thermal excesses associated with the CMB and mantle plumes.

Rates of entropy production are very sensitive to the present-day core/mantle heat flow. To maintain roughly equal differences in temperature across the lower thermal boundary layer for the simulations in **Fig. 5**, we vary $d_s$ from 72 km for $Q_{CMB}(t_0) = 5 \text{ TW}$ to 38 km for $Q_{CMB}(t_0) = 15 \text{ TW}$ in equal increments of 12 km per 5 TW. If $Q_{CMB}(t_0)$ is $\sim 5 \text{ TW}$, at the lower end of modern estimates, then even $C_M = 5 \times 10^{-5} \text{ K}^{-1}$ is insufficient to sustain positive dissipation. Without precipitation, $E_\theta \approx 100-150 \text{ MW K}^{-1}$ before the inner core nucleates if $Q_{CMB}(t_0) = 15 \text{ TW}$, $[K] = 0 \text{ ppm}$, and $H_{eff} = 300 \text{ kJ mol}^{-1}$. However, higher dissipation rates are likely required to produce the present-day magnetic field strength (Nimmo, 2015). Decreasing $Q_{CMB}(t_0)$ implies that values of $Q_{CMB}$ are depressed by roughly the same amount in the past and also that the inner core is older.

Increased abundances of potassium in the core imply lower rates of ohmic dissipation in the past. This result may seem counterintuitive because radiogenic heating is a positive source of energy and entropy—allbeit an inefficient one because of another Carnot-like efficiency term. Calculations that consider only the thermal evolution of the core demonstrate that increased radioactive lowering the amount of secular cooling required to sustain a dynamo and thus the temperature of the core in the past, but also necessitates a higher core/mantle heat flow (e.g., Nimmo, 2015; Labrosse, 2015; O’Rourke and Stevenson, 2016). There is no reason, however, that $Q_{CMB}$ should increase because of radiogenic heating in the core. In fact, the relatively slow cooling implied by such heating tends to decrease $Q_{CMB}$ by lowering the temperature contrast across the core/mantle boundary. Fundamentally, the rheology of the lower mantle governs $Q_{CMB}$. Any given value of $Q_{CMB}$ will yield more entropy for a dynamo if cooling causes inner core growth or precipitation rather than just removing heat from the decay of potassium or other radioactive isotopes of uranium and thorium.

We also repeated these sensitivity tests using the lower bound on thermal conductivity. With $k_c(0) = 40 \text{ W m}^{-1} \text{ K}^{-1}$, positive dissipation is always maintained for each value of $H_{eff}$, $Q_{CMB}(t_0)$, and $[K]$ even absent precipitation. Low thermal conductivity also permits $E_\theta \approx 400 \text{ MW K}^{-1}$ when $Q_{CMB}(t_0) = 5 \text{ TW}$ and $C_M = 4 \times 10^{-5} \text{ K}^{-1}$. If $Q_{CMB}(t_0) = 5 \text{ TW}$ with low thermal conductivity but no precipitation, then $E_\theta \approx 100 \text{ MW K}^{-1}$ in the past and rises to $\sim 300 \text{ MW K}^{-1}$ when the inner core nucleates at $\sim 1.4 \text{ Gyr}$ before today. Increasing $Q_{CMB}(t_0)$ to 15 TW then yields $\sim 500$ and 1250 MW K$^{-1}$ of entropy production prior to and following the nucleation of the inner core, respectively.
Fig. 4. Simulations showing that increasing the mantle heat flow implies increased rate of entropy production available for the dynamo in the past (a), potential temperature of the mantle (b), and heat flow from the mantle to the surface (c), while the Urey ratio (d) is decreased.

Fig. 5. Simulations with $H_{ij}$ varied from $-300$ to $300$ kJ mol$^{-1}$ (left), $Q_{CMF(t)}$ from 5 to 15 TW (center), and $[K]$ from 0 to 200 ppm (right). Top: Rate of entropy production available for the dynamo. Bottom: Heat flow across the core/mantle boundary.

4. Discussion

4.1. Earth's initially hot state

The final stage of Earth’s formation featured a number of violent collisions, notably including the Moon-forming impact, that would have at least partially melted the mantle (e.g., Rubie et al., 2015). Even absent giant impacts, the gravitational energy associated with accretion is large enough to create temperatures in the core and mantle much higher than those prevailing today. Without high temperatures at the time of core formation, insufficient magnesium would partition into the core to provide an appreciable amount of compositional buoyancy during its later evolution (Wahl and Militzer, 2015; O’Rourke and Stevenson, 2016; Badro et al., 2016). If extrapolated backwards until the time of Earth’s accretion, however, our scalings of plate tectonics predict that the mantle was not much hotter than it is today. Efficient heat loss from a vigorously convecting magma ocean must have actually occurred after accretion. We have not explicitly included a period of rapid cooling before the initiation of plate tectonics, so our simulations may not be representative of Earth’s earliest history.

Assuming that plate tectonics operated throughout the Proterozoic is quite reasonable (e.g., Korenaga, 2013), and our central goal
is explaining how Earth sustained a dynamo throughout this eon. Observational evidence for the operation of plate tectonics is lacking for the same reason—a scarcity of rocks—that the existence of a magnetic field in the deep past remains controversial. Calculations in this paper and fully dynamical simulations using the new scaling for plate tectonics suggest that the potential temperature of the mantle was $\sim 1800$ K at $>3$ Ga (Herzberg et al., 2010; Korenaga, 2011). Crucially, this is within $\sim 100$ K of the potential temperature necessary for a surface magma ocean (e.g., Rubie et al., 2015), meaning that our simulations of plate tectonics should smoothly connect with models that describe the solidification of a magma ocean and possibly another, short-lived regime of mantle convection (e.g., Moore and Webb, 2013).

A magma ocean extending from the surface through the transition zone to $660$ km depth would have existed when the potential temperature was $\sim 2200$–$2300$ K (e.g., Rubie et al., 2015). Subsequently cooling the mantle by $\sim 400$ K over $1$ Gyr, for example, to the state when plate tectonics may have begun requires $Q_M \sim 100$ TW from the magma ocean, more than twice the heat flow associated with solid-state convection. Of course, the actual lifespan of the surface magma ocean, which could be much shorter than $1$ Gyr, is very uncertain (e.g., Solomatov, 2007). Additionally, the temperature of the core may have decreased by $\sim 1000$ K or more during the earliest phase of cooling after Earth’s “hot start.” After this initial burst, a long-lived magma ocean at the base of the mantle may have delayed the onset of the geodynamo. That is, the core would not continue cooling below the liquidus temperature of the mantle melt until the basal magma ocean solidified (Labrosse et al., 2007). Discovering whether a global magnetic field existed throughout the Archean and Hadean would provide critical constraints on these processes.

4.2. Limitations of our modeling approach

Using one-dimensional scaling laws to describe the coupled evolution of Earth’s core and mantle is computationally efficient and allows for rapid sensitivity tests and description of first-order phenomena (e.g., Stevenson et al., 1983; Christensen, 1985). Future work, however, should address some shortcomings of our approach. Parameterizations of core energetics coupled to fully dynamical simulations of the mantle (e.g., Nakagawa and Tackley, 2010) should include precipitation of magnesium-bearing minerals. If CMB heat flow is sub-adiabatic and no compositional buoyancy is available (e.g., before the nucleation of the inner core absent precipitation), then additional equations are required to model the dynamics in the presence of a thick, stable layer at the top of the core since only part of the outer core would vigorously convect (e.g., Labrosse, 2015). We have neglected this complication because such scenarios are probably not compatible with the observed longevity of the global magnetic field.

More importantly, the CMB is both spatially and temporally heterogeneous in terms of composition and temperature. The stagnant layer in our models condenses vertical and lateral variations such as the post-perovskite phase transition and double-crossings, along with regions like large low-shear-wave-velocity provinces and ultralow-velocity zones (e.g., Hernlund and McNamara, 2015). The spatial variability of CMB heat flow caused by cold slabs, in particular, may control the timing of geomagnetic reversals (e.g., Olson et al., 2013). Fluid motions associated with baroclinic instability can drive lateral transport of heat and assist the operation of a dynamo. The associated entropy production, however, is likely small because the effective temperature of dissipation is close to that of the CMB, reducing its Carnot-like efficiency (e.g., Labrosse, 2015). Thus, we have not included any parameterization of this process.

4.3. Implications for Venus

Spacecraft have constrained the magnetic moment on Venus to less than $10^{-5}$ times the terrestrial value (Phillips and Russell, 1987). Although its moment of inertia is presently unknown, assuming that Venus has an iron-rich core like Earth seems reasonable. Thermal evolution models imply that the core of Venus would not have frozen completely solid (e.g., Stevenson et al., 1983), but convection must have ceased in the liquid portion for some reason. Jacobson et al. (2015) proposed that Venus did not suffer a giant impact, in which case a stable stratification would develop as the concentration of light elements in material added to the top of the core increased with pressure/temperature conditions during accretion. No giant impact also means no magnetism to precipitate and provide energy and entropy for the dynamo. Future work should consider estimates of the CMB heat flow from, for example, thermal evolution models (e.g., Nimmo, 2002; O’Rourke and Korenaga, 2015) and the buoyancy flux of mantle plumes (e.g., Smrekar and Sotin, 2012). Estimates above the critical value required to drive a dynamo in a mostly isentropic and homogeneous core would serve as evidence that the core of Venus was indeed initially stratified.

5. Conclusions

Simple scalings for mantle dynamics, along with a parametrized model for the energetics of the core, allow us to estimate how much entropy has been available to sustain a dynamo throughout geologic time. If the recent upward revision of the thermal conductivity of the core is correct, then the precipitation of magnesium-bearing minerals at rates suggested by O’Rourke and Stevenson (2016) and Badro et al. (2016) allows vigorous convection prior to the nucleation of the inner core for most combinations of initial conditions. Ongoing precipitation would produce positive rates of entropy production for at least $3.45$ Gyr as long as the abundance of potassium is under $\sim 200$ ppm and the present-day CMB heat flow is above $\sim 5$ TW. Because the minimum required heat flow across the core/mantle boundary remains roughly constant, the longevity of the magnetic field is compatible with a weak dependence of mantle heat flow on temperature. Precipitation may yield roughly constant rates of entropy production over time, meaning that inner core’s formation may not create a dramatic increase in field strength preserved in the paleomagnetic record. Similar computational exercises are relevant to Venus and probably differentiated “super-Earth” exoplanets.

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Appendix A. Derivation of $Q_P$

The release of gravitational energy associated with the precipitation of magnesium-bearing minerals is easily defined if the rate of precipitation is roughly constant (O’Rourke and Stevenson, 2016)

$$Q_P = -\int_0^\infty \rho(q)\psi'(q)\beta M C_M \frac{dT_{CMB}}{dt} dV,$$

where $C_M$ is the fraction of the core’s mass precipitated per $1$ K of cooling and $\rho$ is the density profile in the core. Since precipitation only occurs out of the liquid portion of the core
\[ Q_p = -\int_{D_C} \rho(r) \psi(r) dV - M_{OC} \psi(R_1) \] \\
\[ \beta_M = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial \xi} \right)_{p,T}, \]

where \( \xi \) refers to the outer core and \( R_1 \) is the radius of the inner core.

The coefficient of compositional expansion associated with magnesium precipitate is

\[ \beta_M = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial \xi} \right)_{p,T}, \]

where \( \xi \) is the concentration of magnesium-rich material. Assuming that the density change between the precipitate and the residual core alloy is roughly equal to that across the core/mantle boundary, then \( \beta_M \approx 0.8 \). This value is probably only accurate within several tenths of percent, but our uncertainty about the value of \( C_M \) is much larger and degenerate.

The gravitational potential in the core relative to zero potential at the CMB is

\[ \psi(R) = \left[ \frac{2}{3} \pi G \rho_0 r^2 \left( 1 - \frac{3}{10} \frac{r^2}{L_p^2} - \frac{A_p}{7} \frac{r^4}{L_p^4} \right) \right] R \left. \right|_{R_C} \]

where \( \rho_0 \) is the central density and \( A_p \) and \( L_p \) are defined in Labrosse (2015) based on the equation of state of liquid core alloy. Since this and the density profile are both available as polynomials, we can write an analytic equation

\[ Q_p = -\beta M_{CM} \left( \frac{8}{3} \pi G \rho_0 \right) \left[ -\frac{\Gamma}{\bar{r}^3} + \frac{2}{10} \left( 1 + \frac{\Gamma}{L_p^2} \right) \right] \]

\[ + \frac{1}{71 \bar{r}^2} \left( \frac{A_p \Gamma - 13}{L_p^3} \right) \frac{10}{\bar{r}^2} + \frac{1}{\bar{r}^4} \left( \frac{9}{7} \right) \frac{8 A_p}{7} \frac{r^4}{ \bar{r}^4} \]

\[ + \frac{3 A_p}{770 \bar{r}^5} \frac{11}{13} \left( \frac{91}{1} \right) R \left. \right|_{R_C} \]

\[ - \Gamma M_{OC} \frac{d \psi_{CMB}}{d \bar{r}} \left. \right|_{R_1} \]

where

\[ \Gamma = R^2 \left( 1 - \frac{3}{10} \frac{L_p^4}{r^4} - \frac{A_p}{7} \frac{L_p^4}{r^4} \right). \]


